Design Research through Lesson Study & Pedagogical Tools in Mathematics

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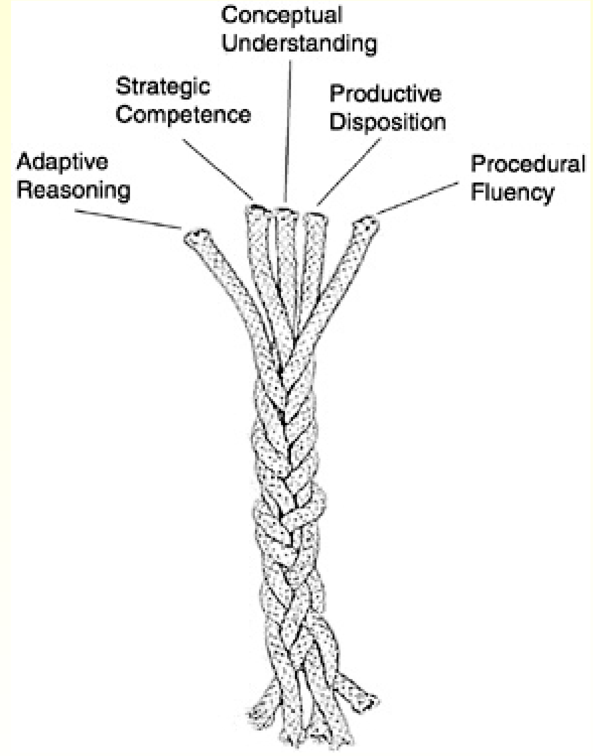
Mathematical proficiency

- ***Conceptual understanding*** **-** comprehension of mathematical concepts, operations, and relations
- ***Procedural fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive reasoning*** **-** capacity for logical thought, reflection, explanation, and justification
- ***Productive disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.



Adding it Up (National Research Council, 2001)

Developing highly qualified mathematics teachers by…

Developing Pedagogical Content Knowledge :

1. subject matter knowledge,
2. pedagogical knowledge,
3. knowledge of context.

Mathematical Teaching Knowledge

" Teachers need to know and understand mathematics in ways directly related to the work of teaching, for example, designing good tasks, diagnosing the difficulties that students are having and managing a productive discussion of mathematics in class." (Sztajin, Ball and Mcmahon, 2006)

Why is modeling mathematics concepts important in developing pedagogical content knowledge ?

PEDAGOGICAL CONTENT KNOWLEDGE is knowing the

“most useful forms of representation of these ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others " (Shulman, 1987, p. 9).

Design Research through Lesson Study

Lesson Study- Japanese teacher led professional development model that promotes teachers as researchers and develop PCK

Iterative cycle of design-analysis and redesign cycles leading to improvements in teaching & learning

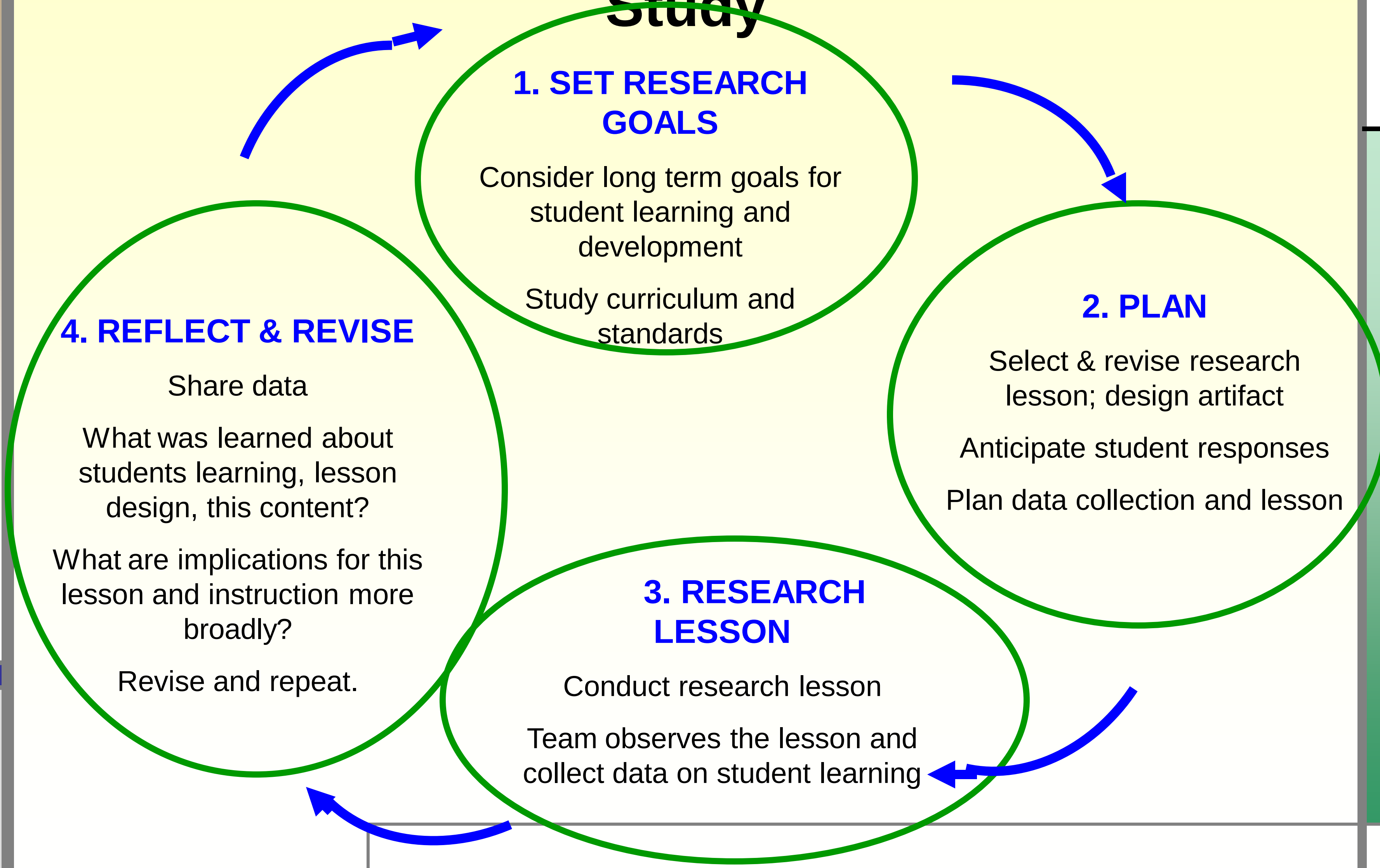
Collaboration (Researchers & Practitioners)

Retrospective analysis (debrief)

Design Research-

attempt to “engineer innovative educational environments”

Design Research through Lesson Study

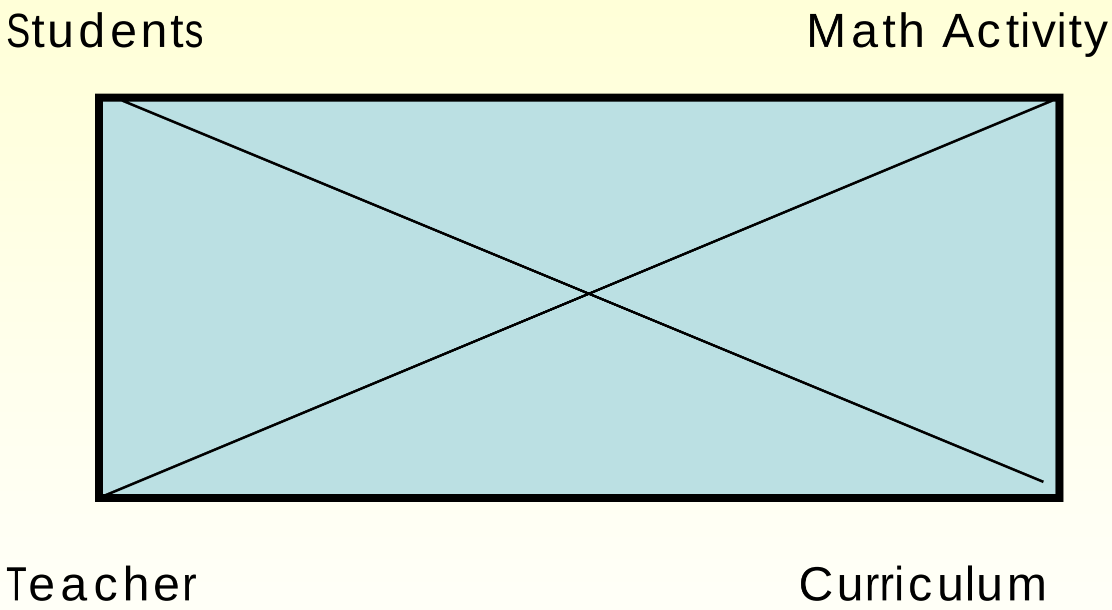


Pathways to Instructional Improvement

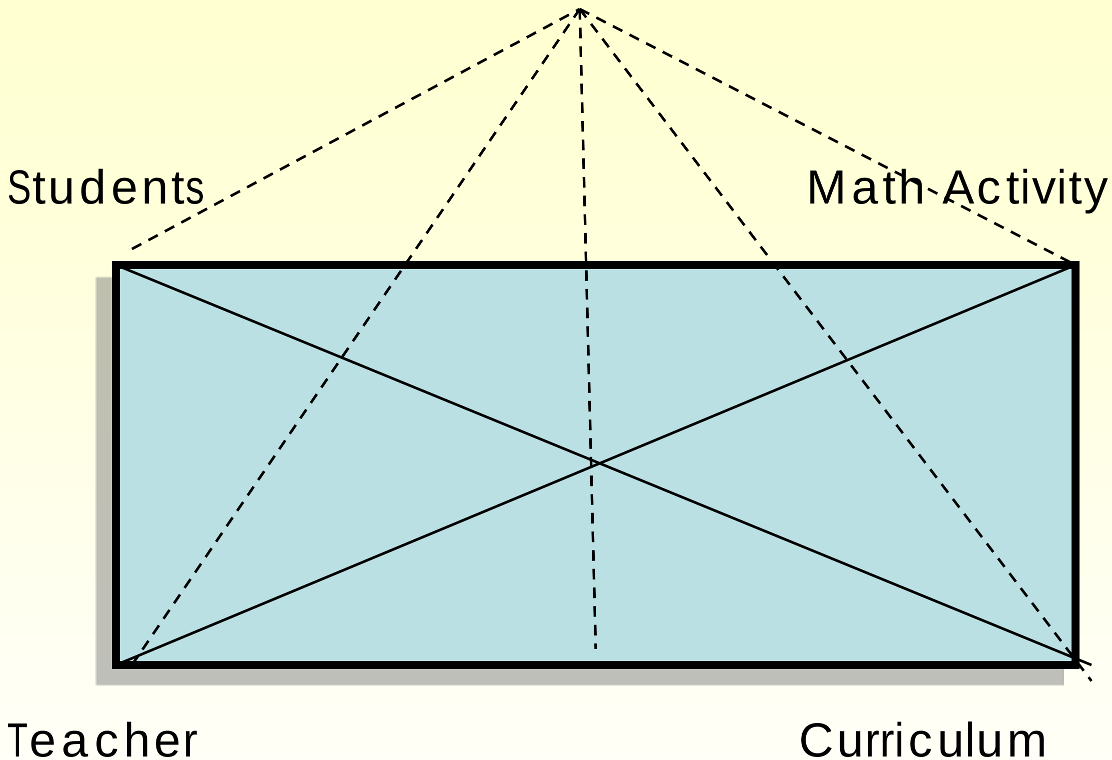
- Increased content knowledge
- Increased knowledge of instruction
- Increased ability to observe students
- Build stronger collegial networks
- Connect daily practice to long-term goals
- Increase motivation and self-efficacy
- Improve quality of lesson plans

(Lewis, Perry, & Hurd, 2004)

Pedagogical Tools in Mathematics & Lesson Study

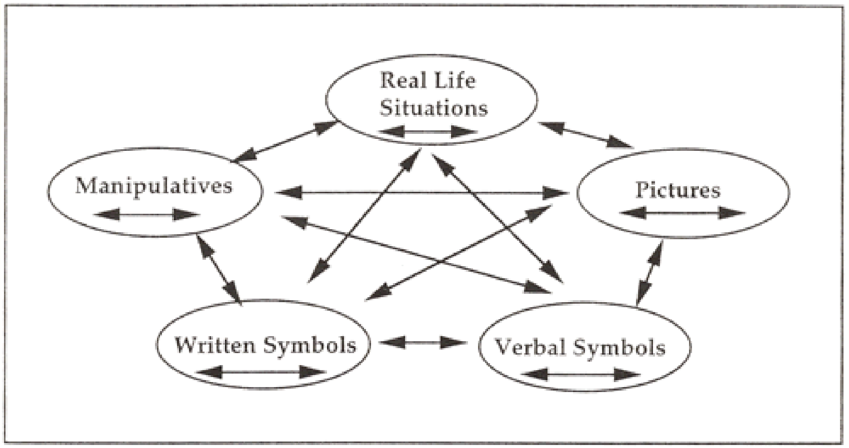


Pedagogical Tools



Pedagogical Tools

- Manipulatives (Physical and Virtual)
- Discourse
- Writing
- Contextual Problems
- Analogies
- Concept maps



Importance of mathematical models

- "Models *of* thinking into models *for* thinking" (Gravemeijer 1999, 2000)
- Helps students build a network of mathematical relationships and make generalizations
- Gives access to students to form mental images of concepts and context

REPRESENTATIONS-Defining teachers and students' use

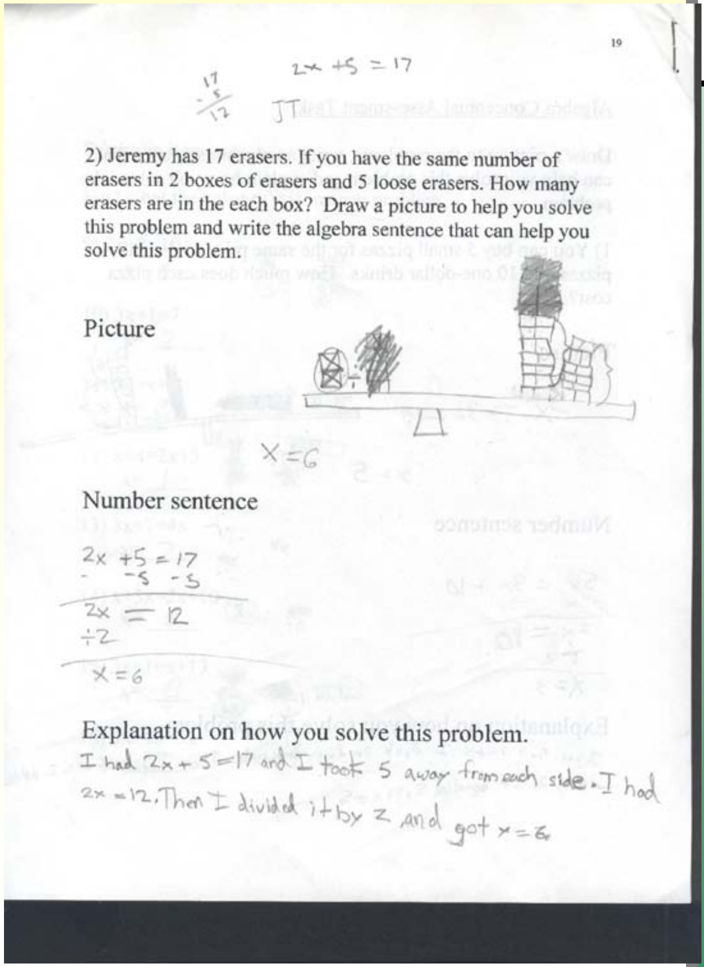
Both presentational model

(used by adults in instruction)

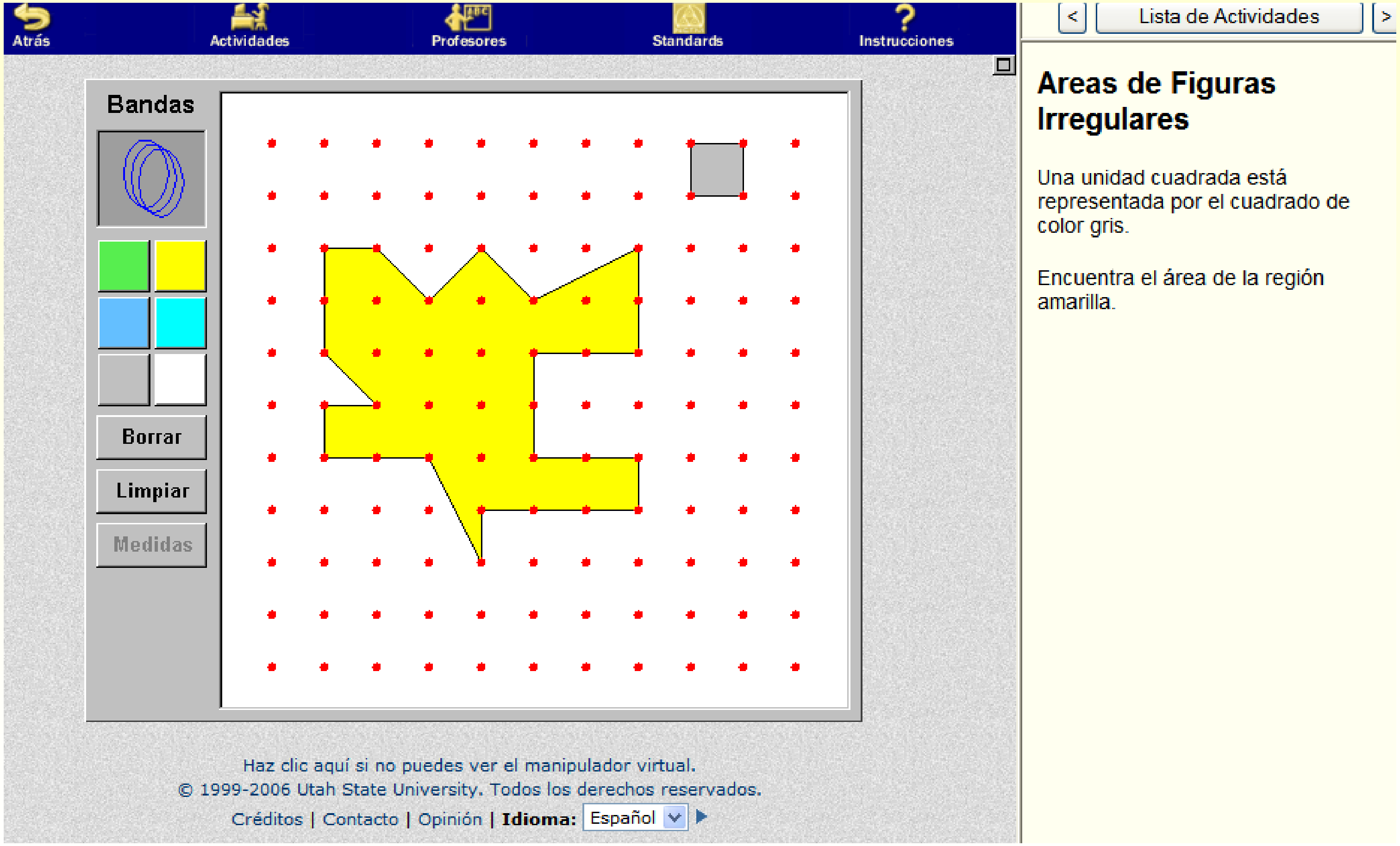
Re-presentational model

(produced by students in learning)

(Lamon, 2001)

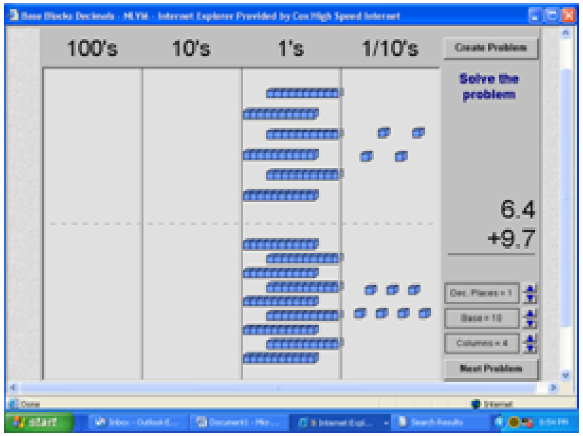


…from tools to represent thinking into models *for* thinking



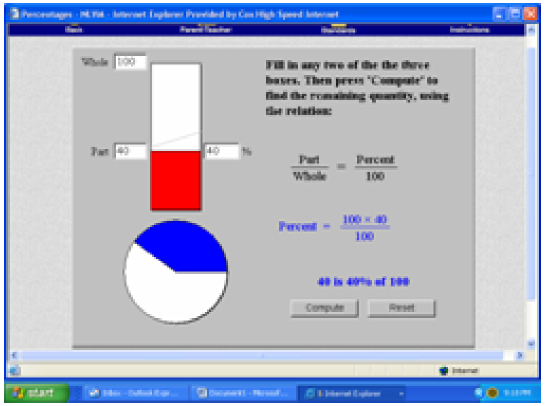
…build a network of mathematical relationships

Model A: Working with place value – decimals to 1/10th

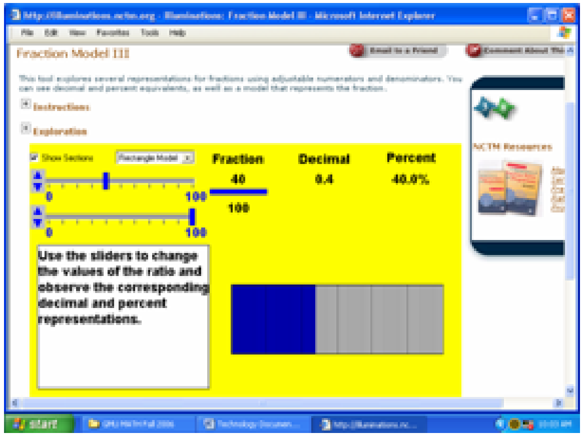


Model B: Fractions

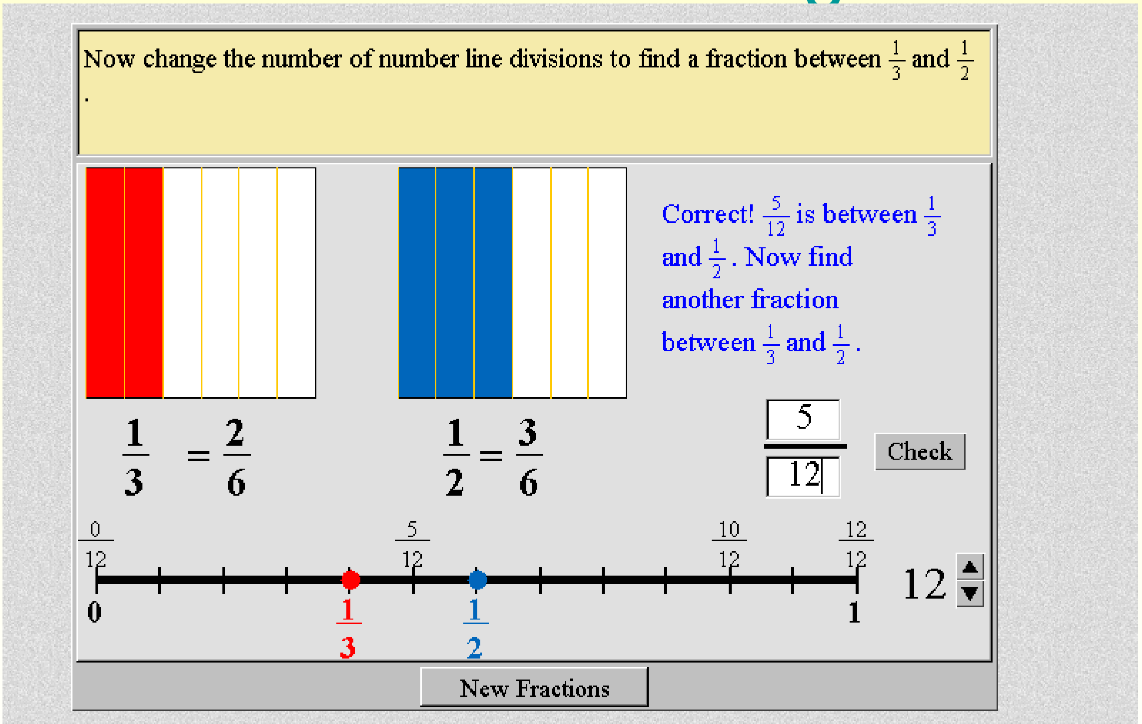
Model C: Fractions in Base 100 convert to percentages



Model D: Fraction Model 3 – Converting between fractions, decimals and percentages



…build deeper understanding through multiple representations & mental images



Example of Mathematical Model to teach density of rational numbers

Challenge 1: Teachers' inability to model mathematics concepts stems from having procedural knowledge without conceptual understanding

- Although 43% of the US teachers successfully calculated 1 ¾ / ½ , almost all failed to come up with a representation of division by fractions.
- Among 23 teachers, 6 could not create a story and 16 made up stories with misconceptions.

From Liping Ma's (1999), *Knowing and Teaching Mathematics*

Challenge 2: Teachers need profound understanding of fundamental mathematics.

PUFM goes beyond being able to compute correctly and to give a rationale for computational algorithms…

… aware of the conceptual structure and basic attitudes of elementary mathematics and is able to teach them to students

From Liping Ma’s (1999), *Knowing and Teaching Mathematics*

Challenge 3: Inappropriate use of mathematics tools (manipulatives and technology

"Magical hopes: Manipulatives and the reform of math education." (Ball, 1992)

"Manipulatives Don't Come with Guarantees" (Baroody, 1989)

Technology for technology sake

Challenge 4: Teachers need to experience effective technology integration in content areas.

“A majority of teacher preparation programs are falling far short of what needs to be done…colleges and universities are making the same mistake that was made by K-12 schools; they treat ‘technology’ as a special addition to the teacher education curriculum

National Council for Accreditation of Teacher Education (NCATE, 2001, p. 7)

Five important consideration for technology integration

(Garofalo, et al., 2000, p. 66)

- Introduce technology in context
- Address worthwhile mathematics with appropriate pedagogy
- Take advantage of technology
- Connect mathematics topics
- Incorporate multiple representations

Research Question…

- How does mathematical modeling and representations of virtual manipulative facilitate the development of pedagogical math content knowledge?

- What impact does technology tools have in teaching and learning?

Methods:

Participants:

Twenty-one pre-service teachers in Elementary Mathematics Methods classroom

Data sources:

- Teacher surveys with likert scale & open ended questions
- Artifacts from lesson study collaboration
- Modeling Mathematics Online activity
- Analysis of Technology/ Mathematics lesson plans

Procedure: Mathematical modeling via techtools

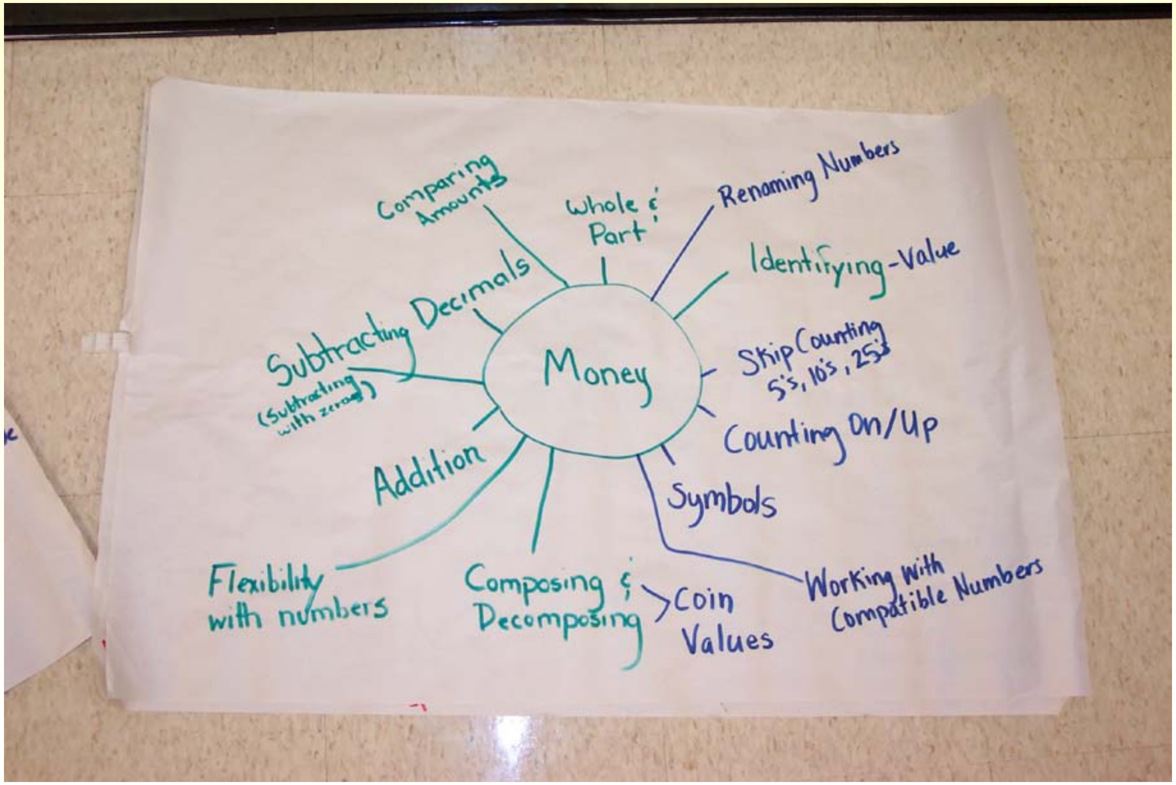
Process 1: Relearning the mathematics content as a “teacher”

Process 2: Selecting and evaluating for appropriate mathematical models

Process 3: Implementing the mathematics model in a lesson

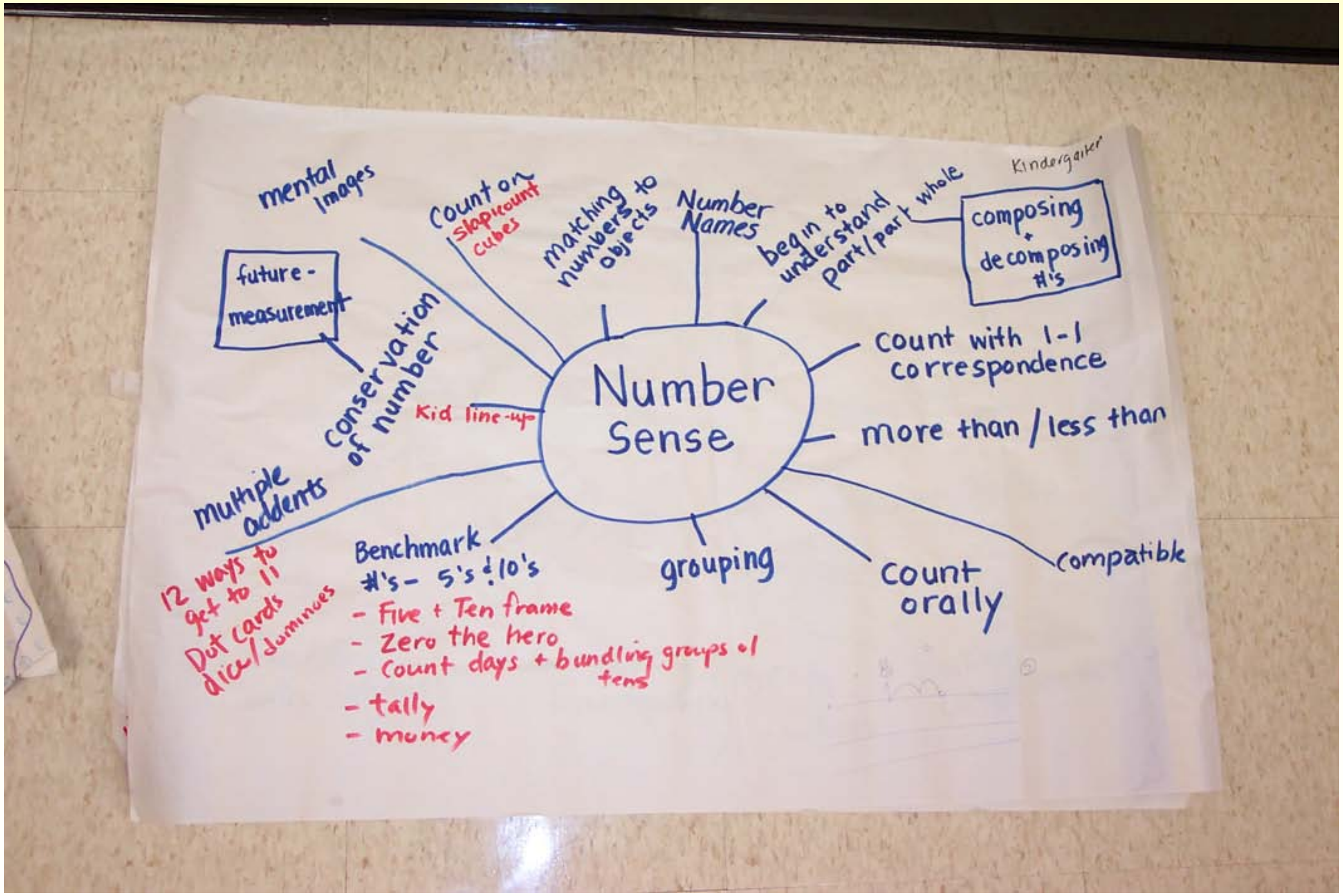
PROCESS ONE : Relearning the mathematics content as a “teacher”

Define the mathematical essence for a lesson using knowledge maps
Developing relational understanding through lesson study collaboration



Knowledge maps display network of interrelated concepts to build teachers’ mathematics knowledge

PROCESS ONE : Relearning the mathematics content as a “teacher”



Knowledge maps define necessary prerequisite understanding and knowledge for which future learning can build on

PROCESS TWO :
Mathematical modeling activities

Step 1: Selecting for appropriate mathematical models developed teachers' pedagogical knowledge base.

Effective Representational Models have…

1) **Transparency:** how easily can the idea be seen through the representation
2) **Efficiency:** Does the representation support efficient communication and use?
3) **Generality:** Does the representation apply to broad classes of objects or concepts?
4) **Clarity:** Is the representation unambiguous and easy to use
5) **Precision:** How close it the representation to the exact value?

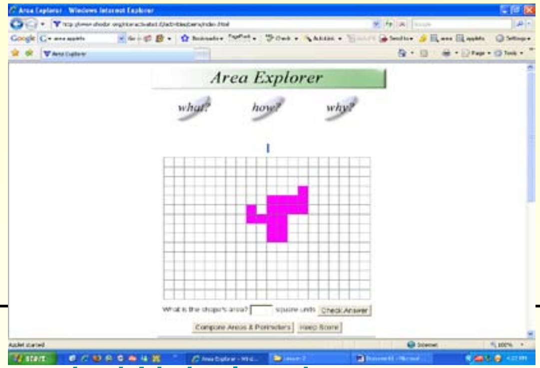
PROCESS TWO : Mathematical modeling activities

Step 2: Evaluating for appropriate mathematical models (math model evaluations) developed teachers' pedagogical knowledge base.

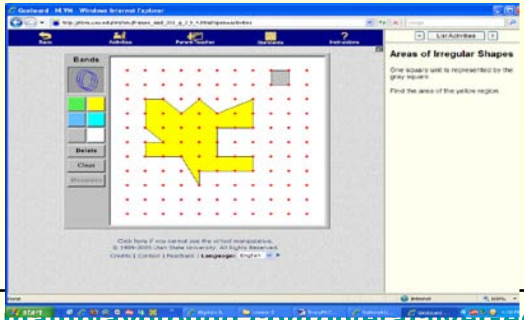
1. **What are the different uses of these models?**
2. **How are students' developmental needs supported by the different uses of these models?**
3. **Is there a developmental progression for these models? Are some used of the model precursors to others?**
4. **What role does the model play in helping students visualize different strategies?**
5. **How might one represent a given strategy with each model?**

Modeling perimeter and area

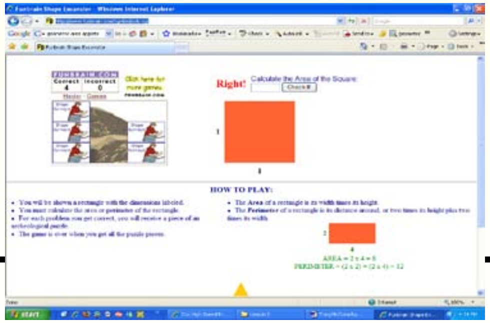
http://www.shodor.org/interactivate1.0/activities/perm/index.html



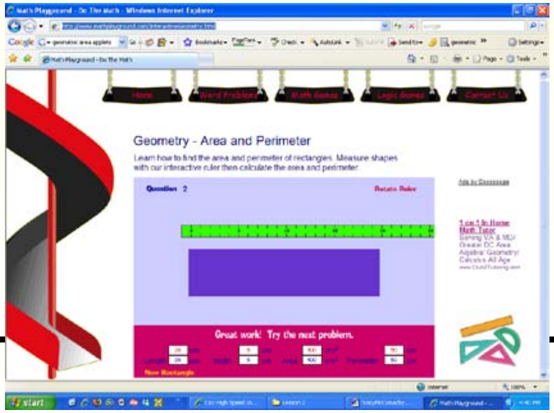
http://nlvm.usu.edu/en/nav/frames_asid_281_g_2_t_4.html?open=activities



http://www.funbrain.com/cgi-bin/poly.cgi

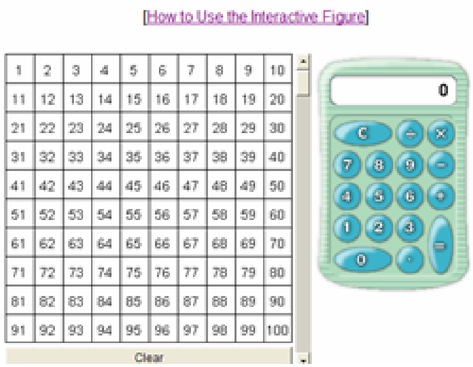


http://www.mathplayground.com/interactiveGeometry.html

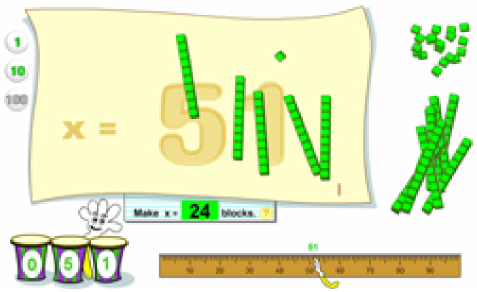


Modeling “ten-ness”

Hundreds chart

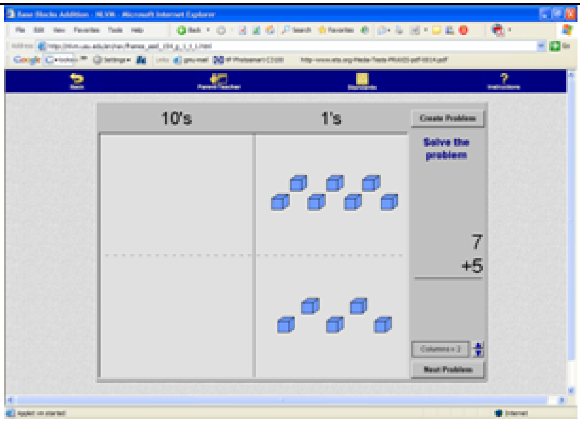


http://standards.nctm.org/document/eexamples/chap4/4.5/index.htm



http://www.learningbox.com/base10/BaseTen.html

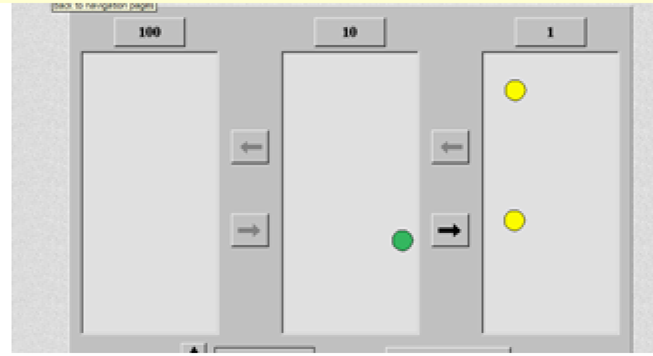
http://www.learningbox.com/base10/CatchTen.html



http://nlvm.usu.edu/en/nav/frames_asid_154_g_1_t_1.html

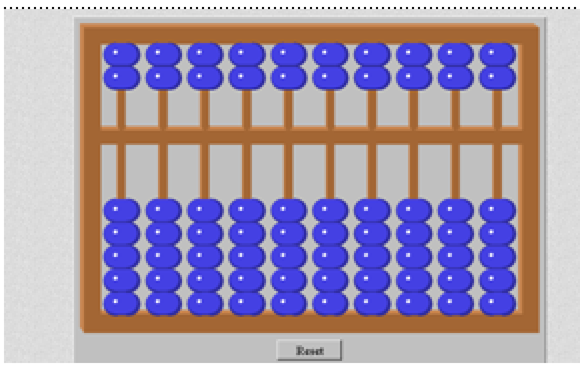
Modeling “ten-ness”

http://www.fi.uu.nl/toepassingen/03106/task1.html



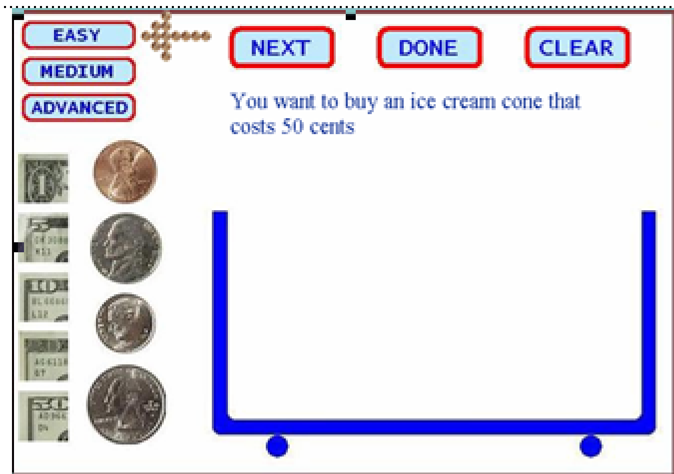
http://nlvm.usu.edu/en/nav/frames_asid_209_g_1_t_1.html?open=activities

Abacus



http://nlvm.usu.edu/en/nav/frames_asid_196_g_2_t_1.html?open=activities

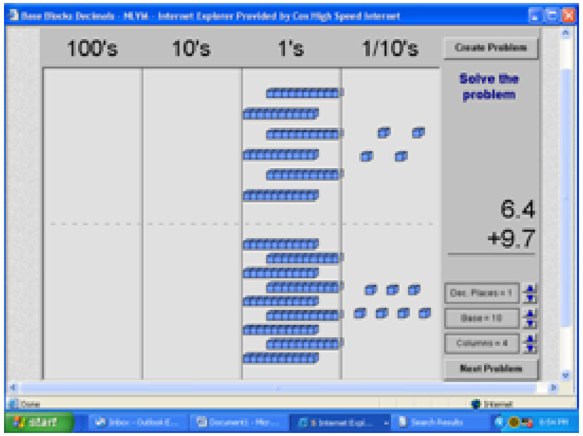
Coins



http://arcytech.org/java/money/money.html

Multiple representations of the concept

Model A: Working with place value – decimals to 1/10th



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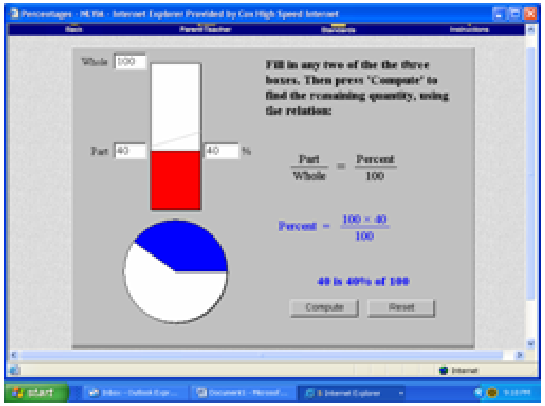

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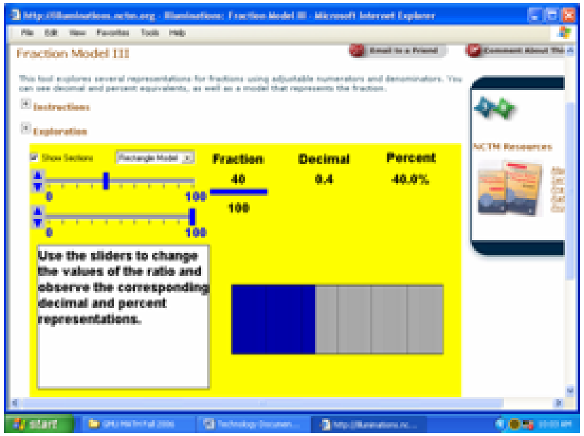


PROCESS THREE :
Implementation phase

Implementing the mathematics model in a lesson

- Concept tutorials
- Guided Investigations/problem solving
- Free Explorations

Results:

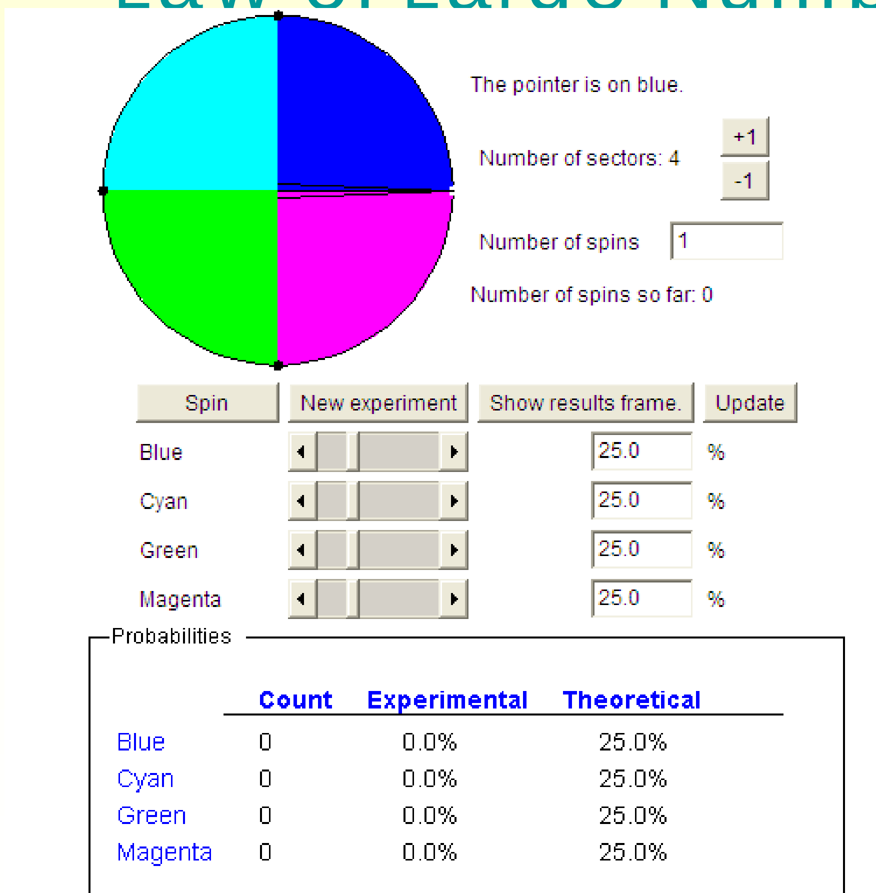
Preservice teachers reported that the mathematical models via virtual manipulatives provided…

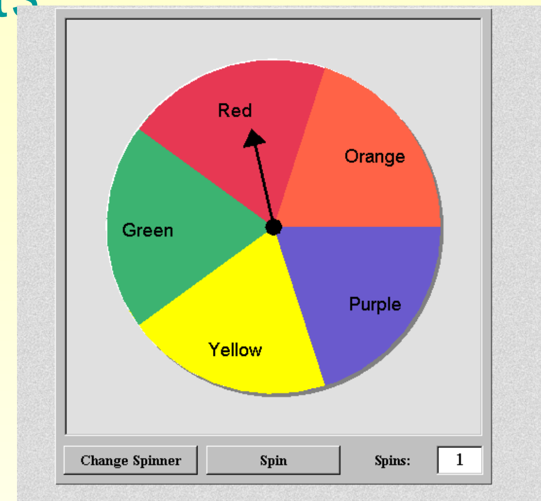
- Visualization that provides a link between concrete and abstract
- Strategy for students to use for collaborative knowledge construction
- New (nontraditional) ways to model math ideas

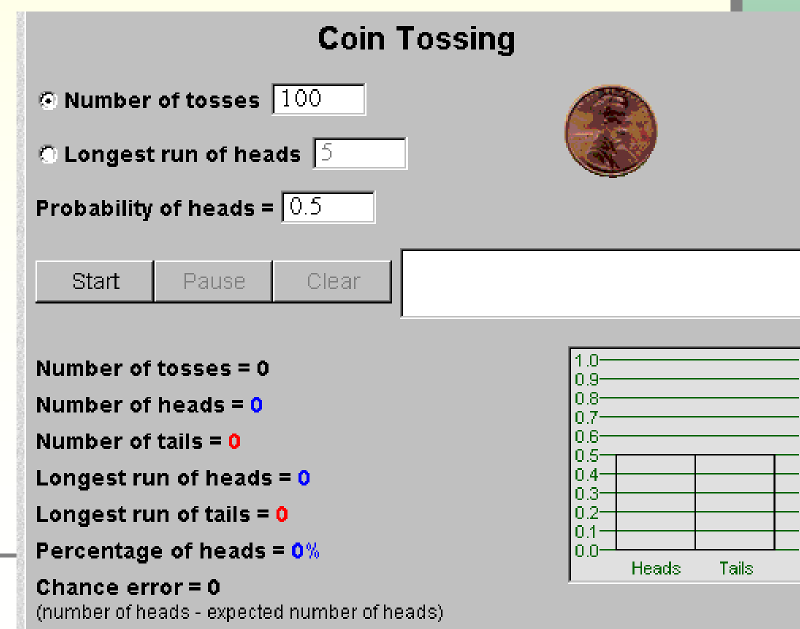
New (nontraditional) ways to model math ideas

Probability

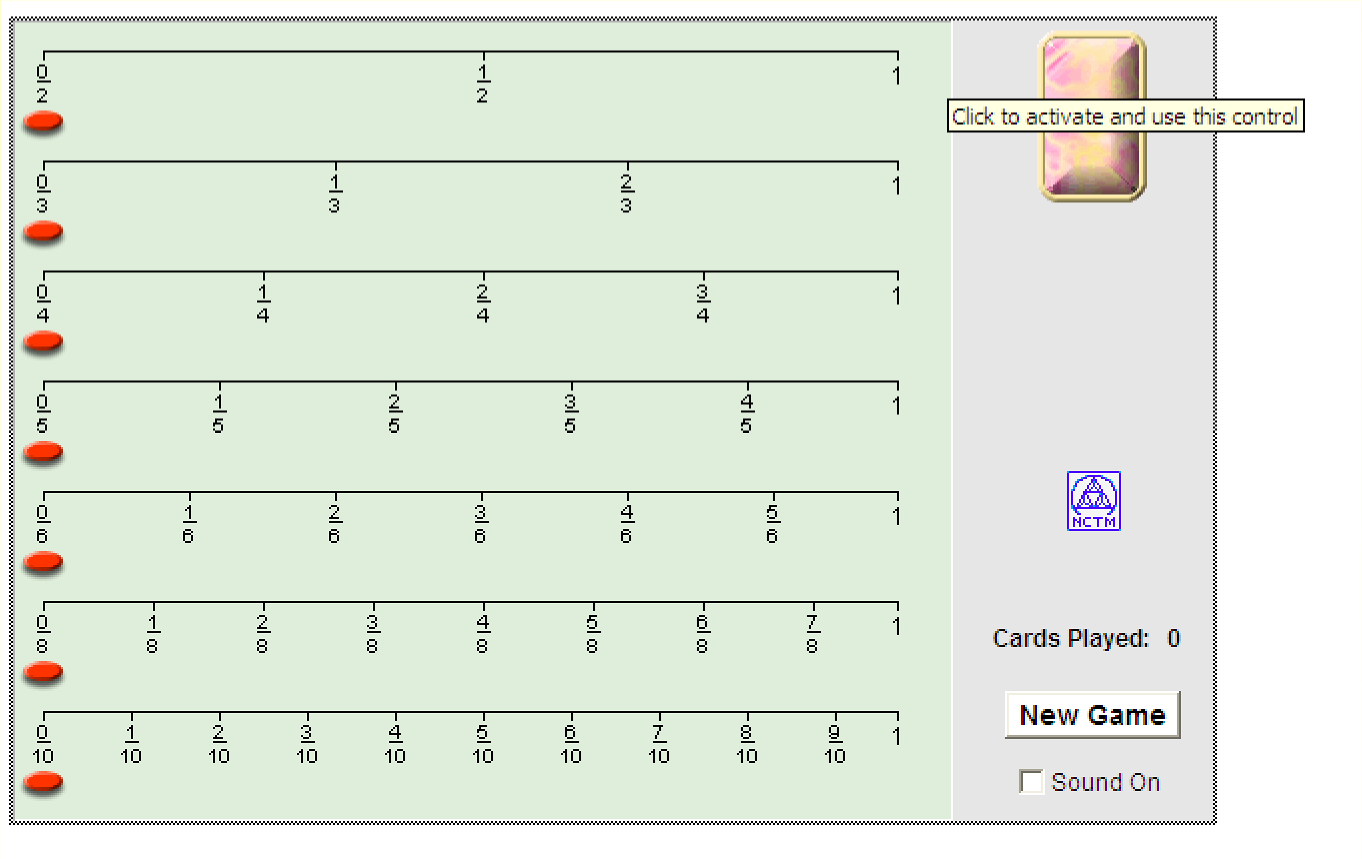
“Law of Large Numbers”







Strategy for students to use for collaborative knowledge construction

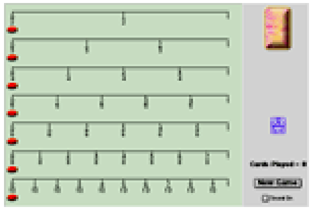
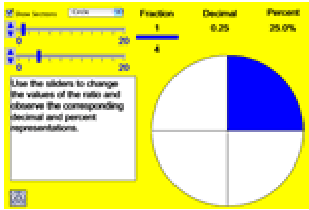
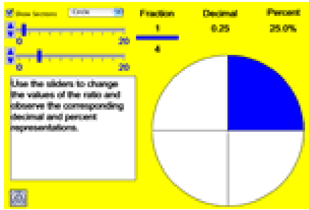
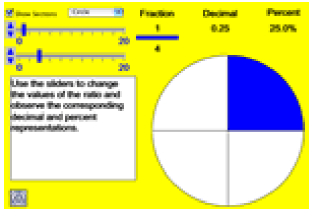


Explore relationships among fractions while playing this interactive game.

Ease of differentiation and tiered learning

These are found in the illumination activity site. Scroll down to fraction

http://illuminations.nctm.org/

	**Fraction Game**		Explore relationships among fractions while playing this interactive game.
	**Fraction Model I**		Explore different representations for fractions. (In this version, the numerator is restricted to values from 0 to 20, and the denominator is limited to benchmark values of 1, 2, 4, 5, 8, 10 and 20.)
	**Fraction Model II**		Explore different representations for fractions. (In this version, the numerator and denominator can both take values up to 20.)
	**Fraction Model III**		Explore different representations for fractions. (In this version, the numerator and denominator can take values up

Results:

Preservice teachers reported that the mathematical models via virtual manipulatives provided…

- User-friendly tasks with visual models for visual learners, and ESOL students who need the support of visuals
- Interactive tools to entice reluctant learners and special ed. students who could not focus on work with manipulatives

Designing lessons using VM

- Record thinking using a tasksheet & print out work for assessment purposes
- Stress the importance of classroom discourse before, during and after using the tools
- Explore relationships and patterns in mathematics using tools
- Use the 5 criteria for selecting representations
- Use to conceptually understand procedural algorithm (ex. Fraction multiplication)

Technology integration survey

Mathematical Models and Technology

	Strongly agree	Agree	Unsure	Disagree	Strongly Disagree
Question 1.Students can learn more mathematics more deeply with the appropriate use of technology.	43%	57%			
Question 2. Technology tools provide visual models that many students are unable to generate independently.	71%	29%			
Question 3. Technology in the mathematics classroom is best used for remediation, or reinforcement of skills.			29%	57%	14%
Question 4. Technology in the mathematics classroom is best used to promote students' analytical, creative, and other higher ordered thinking skills.	14%	29%	29%	29%	
Question 5. Technology allows more time for conceptualizing and modeling mathematical ideas.	29%	57%	14%		
Question 6. Technology offers teachers options for adapting instruction for individual needs of students.	71%	29%			
Question 7. Teachers can learn more ways to model mathematics concepts by using virtual manipulative applets.	57%	43%			

Methods classes and professional development training …

Teachers need to relearn & rethink mathematics considering the critical features:

1. nature of classroom task,
2. role of teacher,
3. classroom culture,
4. mathematics tools and
5. equity and accessibility.

Implications for teacher educators

(1) Within the context of their content and methods courses, pre-service teachers utilize technologies they would use with their own future students;

(2) Meaningful activities and technologies are selected purposefully (not just technology for technology's sake);

(3) Technology integration occurs throughout the teacher preparation program, not just one semester, for example; and

(4) Faculty use technology as a means for modeling and for representing content and pedagogy

Path to pedagogical content knowledge

"American educators assume that you need to know content knowledge before you can plan lessons. Chinese teachers think you learn content knowledge by planning lessons."

from Liping Ma's (1999) Knowing and Teaching Elementary Mathematics

Math-applet Evaluation

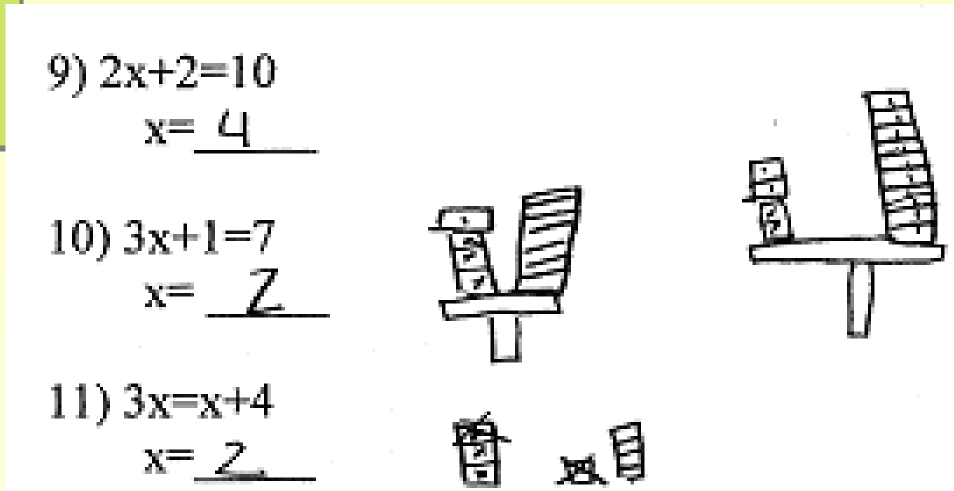
Website	(Exact link)		
Math Strand		Grade level	
Description of mathematical concept (NCTM/SOL)			
Analysis of Mathematical Representations and Models			
__Concept tutorial/Skill Practice __ Investigation/problem solving __Open exploration			
Mathematical thinking opportunities afforded by the mathematics applet			

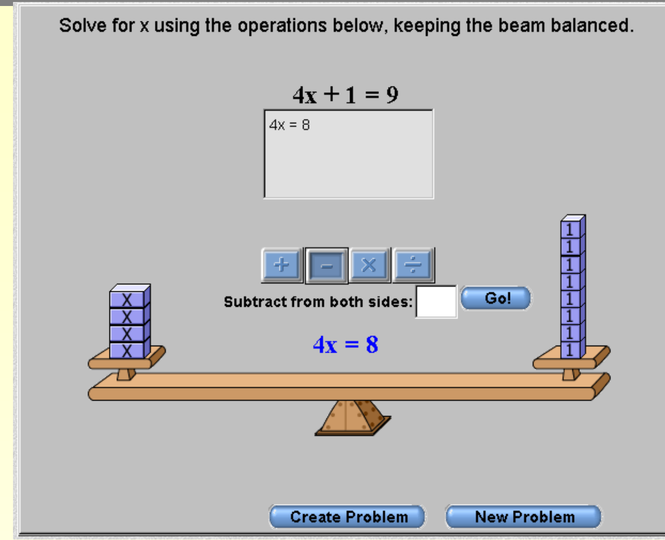
Representations: • Create and use representations to organize, record, and communicate mathematical ideas • Use of mathematical representations to solve problems • Use representations to model and interpret physical, social, and mathematical ideas	Explain:
Communication: • Use the language of mathematics to express mathematical ideas precisely.	Explain:
Connections • Provides connections among mathematical ideas • Relates how mathematical ideas interconnect and build on one another to produce a coherent whole • Apply mathematics in contexts outside of mathematics	Explain:
Reasoning and proof: • Develops reasoning and proof • Opportunities to investigate mathematical conjectures	Explain:
Problem Solving Opportunities to apply and adapt a variety of appropriate strategies to solve problems	Explain:

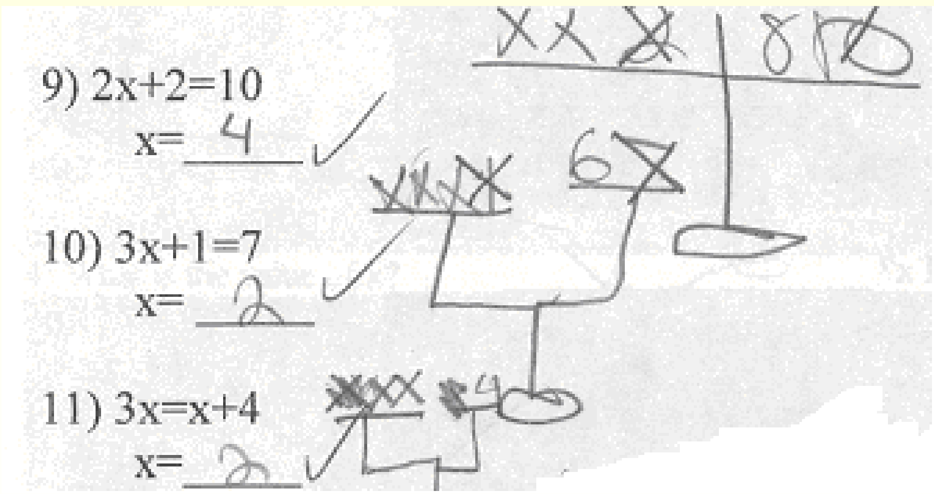
Research focus

1) What mathematical thinking opportunities become amplified by the use of technology tools?

2) What affordances exist in a mathematics technology learning environment that may enhance the learning processes?





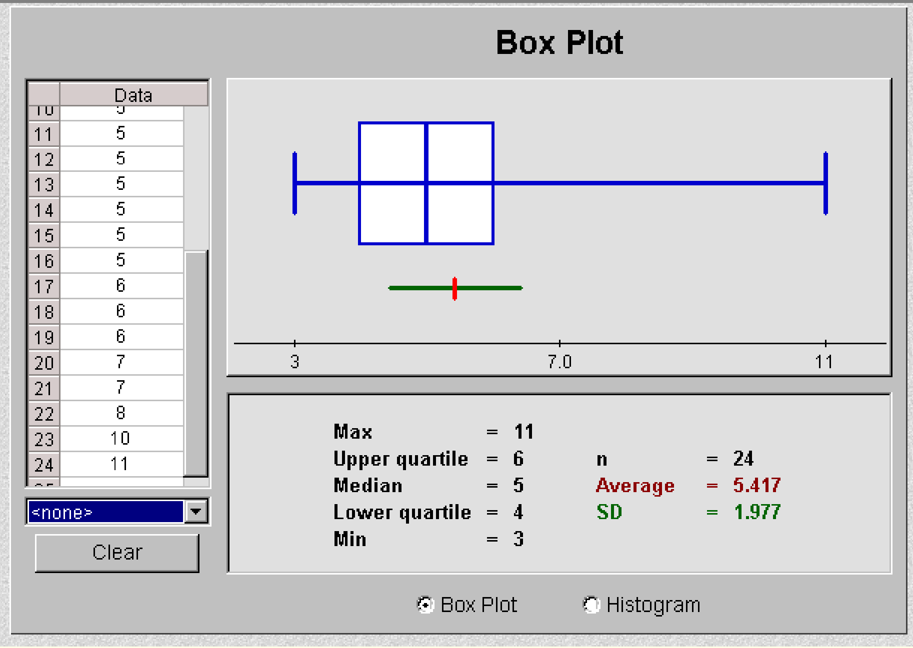


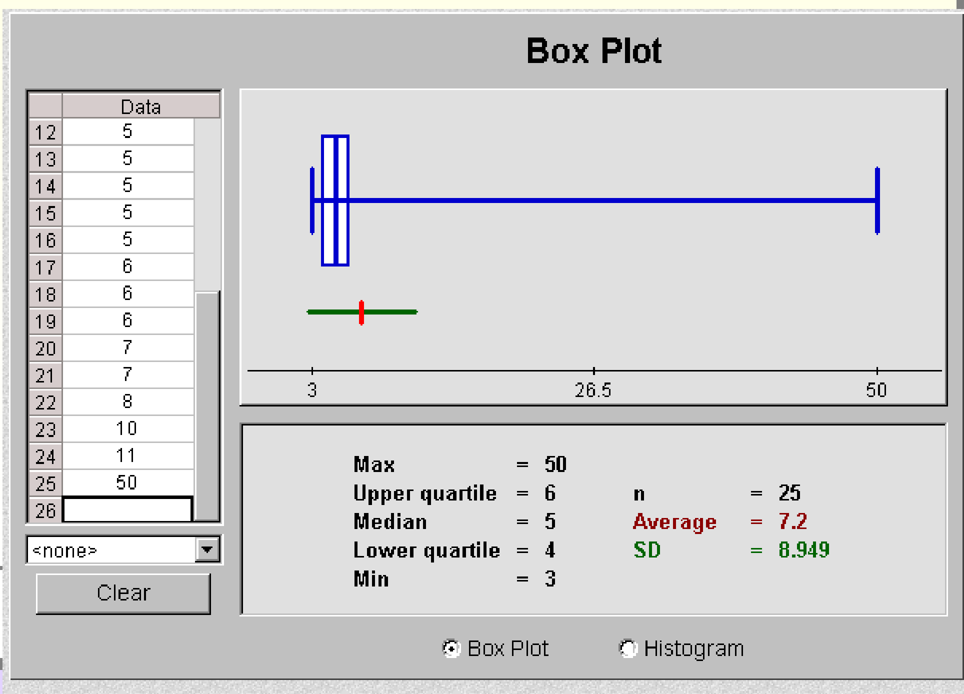
Students' pictorial representations for numeric equations.

The dynamic balance scale in the algebra applet "enforced the mathematical rule of behavior" of the mathematical concept of balancing linear equations.

Making and testing conjectures…

Technology amplified the opportunity available for students to interact with the data to experiment, make conjectures and test out those conjectures to confirm or reject their hypothesis.





Spirit of experimentation