Developing Persistent Flexible Problem Solvers

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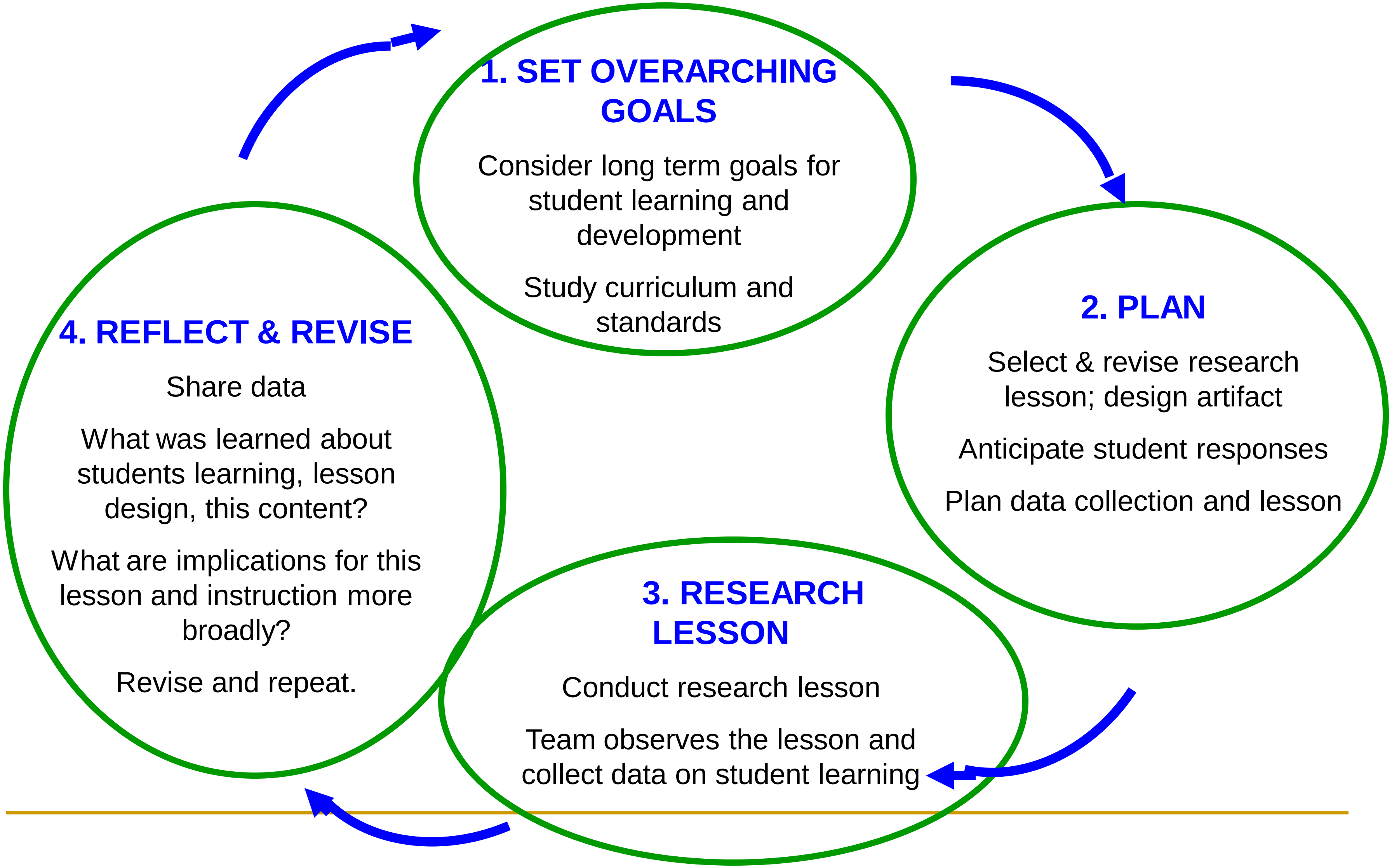
Lesson Study Cycle



“What kind of a problem solver would you like to have in your class?”

Flexibility

- Good problem solvers are flexible and resourceful.

They have many ways to think about problems—
alternative approaches if they get stuck, ways of making progress when they hit roadblocks, of being efficient with (and making use of) what they know.

They also have a certain kind of mathematical disposition—a willingness to pit themselves against difficult mathematical challenges under the assumption that they will be able to make progress on them, and the tenacity to keep at the task when others have given up.

Genius is one percent inspiration and ninety-nine percent perspiration.

EFFORT

- Beliefs about learning and intelligence also influence mathematics performance. When faced with challenging problems, children who believe that intelligence is in large part created by their efforts to learn tend to do better than children who believe that intelligence is a fixed quality that cannot be changed (Dweck, 1999).

From the National Math Panel report 2008

Intelligence = ______ % Ability ______ % Effort

Growth Mindset
Fixed Mindset

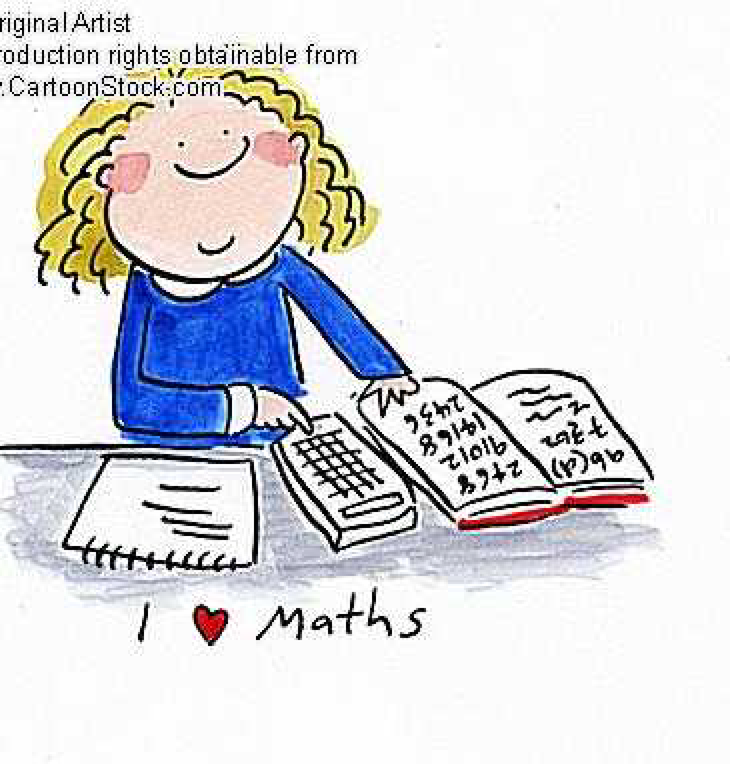
Mindset: The New Psychology of Success (Dweck, 2006)

Fixed Mindset

- Avoids challenges
- Gives up easily
- Sees effort as fruitless or worse
- Ignores useful negative feedback
- Feels threatened by the success of others

Growth Mindset

- Embraces challenges
- Persists in the face of setbacks
- Sees effort as the path to mastery
- Learns from criticism
- Finds lessons and inspiration in the success of others



Self-efficacy

- Student beliefs about the causes of their success and failure have been repeatedly linked to their engaging and persisting in learning activities. Self-efficacy has emerged as a significant correlate of academic outcomes.

- Students can attribute their successes and failures to ability (e.g., I'm just good/bad) at mathematics, effort (e.g., I worked/did-not-work hard enough), luck, or powerful people (e.g., the teacher loves/hates me). These attributions influence students' subsequent engagement in learning.

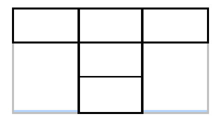
Reflecting on Problem Solving

Clear Communication	Respectful Communication	Flexible Thinking	Persistence
What math words could help us share our thinking about this problem? Choose 2 and explain what they mean in your own words.	Did someone else solve the problem in a way you had not thought of? Explain what you learned by listening to a classmate.	What other problems or math topics does this remind you of? Explain your connection.	What did you do if you got "stuck" or felt frustrated?
What could you use *besides words* to show how to solve the problem? Explain how this representation would help someone understand.	Did you ask for help or offer to help a classmate? Explain how working together helped solve the problem.	Briefly describe at least 2 ways to solve the problem. Which is easier for you?	What helped you try your best? *or* What do you need to change so that you can try your best next time?
If you needed to make your work easier for someone else to understand, what would you change?	What helped you share and listen respectfully when we discussed the problem? *or* What do you need to change so that you can share and listen respectfully next time?	What strategies did you use that you think will be helpful again for future problems?	Do you feel more or less confident about math after trying this problem? Explain why.

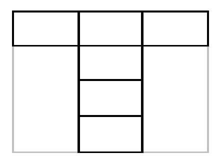
Patterns that Grow!

The Growing T

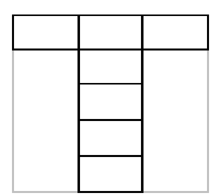
Tommy is building the letter T with cubes. This is what he started with.



Tommy continues to add another cube to his letter T. The next stage looks like this:



He wanted to add another block so that is letter T is growing longer. The third stage looks like this:



Patterns that Grow

1. What would Tommy's T look like if he continues the pattern to the fourth stage?

2. What would Tommy's T look like if he continues the pattern to the fifth stage?

3. What would Tommy's T look like if he continues the pattern to the tenth stage?

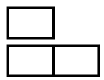
4. What was the pattern you noticed? Explain

5. Create a formula or a rule that you noticed from the pattern.

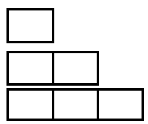
Sara is building a skyscraper with her Legos. She needs to make a staircase to get to the top.

When it was one step high it looked like this:

When it was two steps high it looked like this:



When it was three steps high it looked like this:



How many Legos did she use when the staircase was:

1 step high? ____________

2 steps high? _____________

3 steps high? _____________

4 steps high? _____________

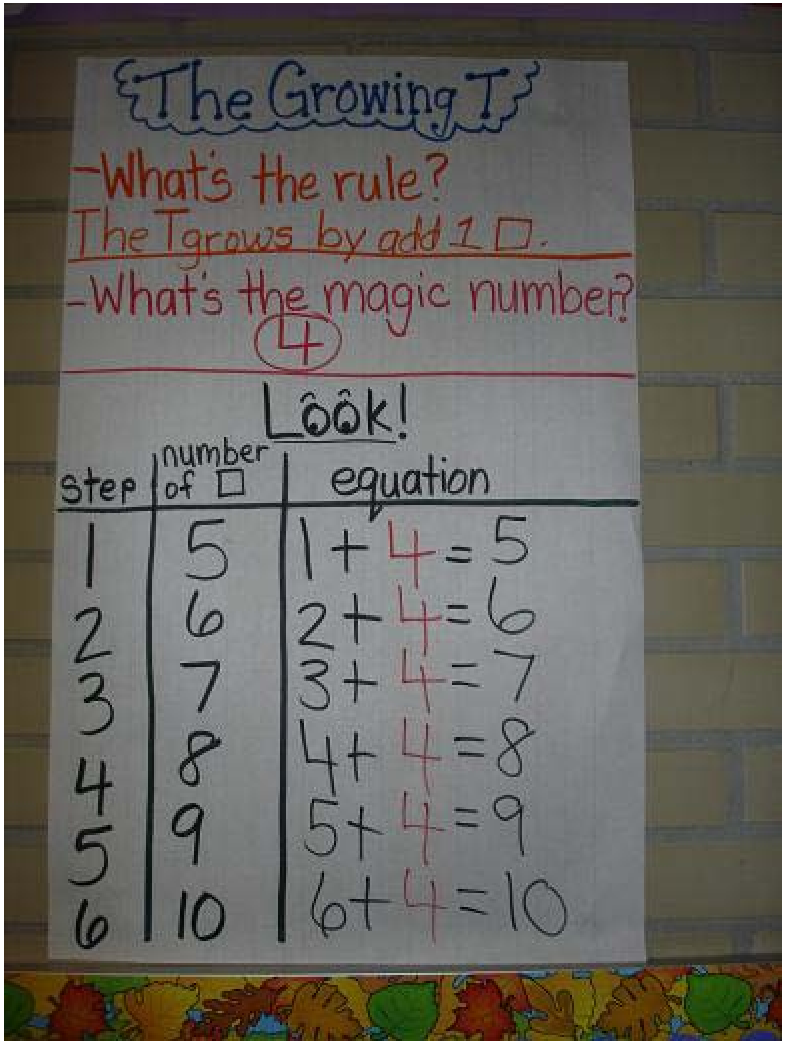
5 steps high? ______________

- What do you notice about the number of Legos used as the staircase gets higher?

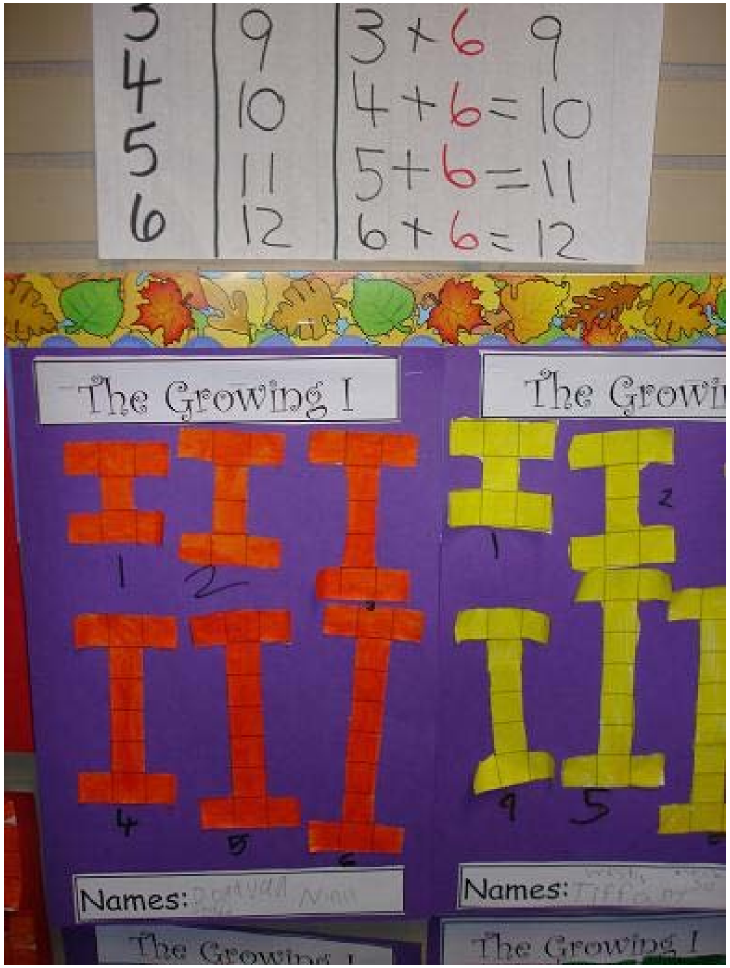
____

- How many Legos will Sara need to make her staircase 10 steps high? (Use the space below to show how you solved this problem. Explain using pictures, numbers, or words).

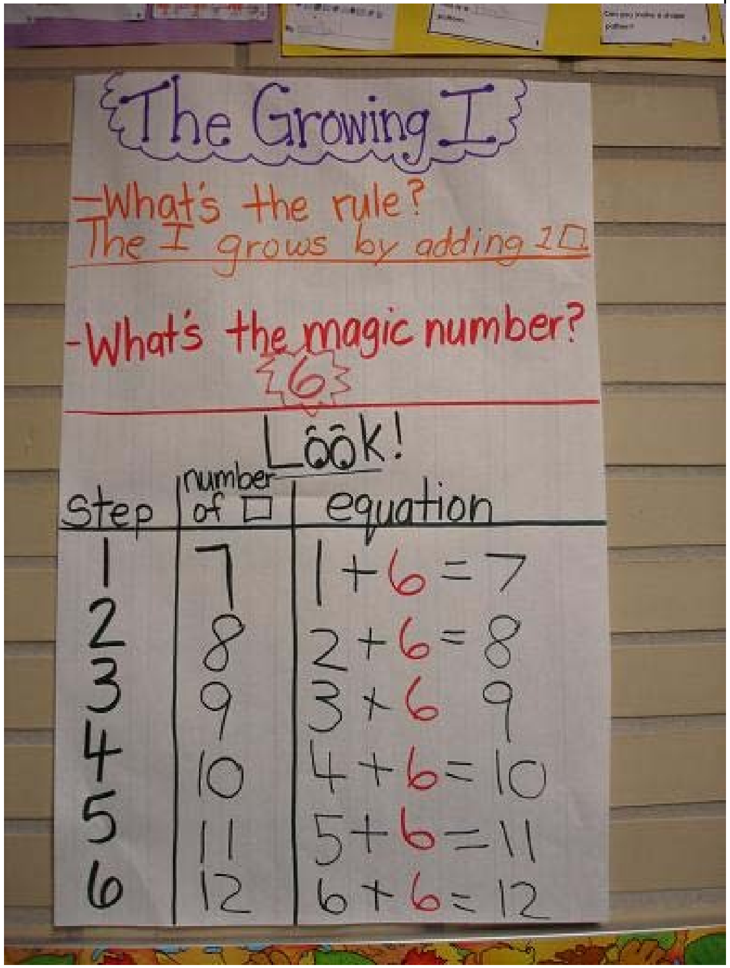
- Number of Legos: ________________
- Show work below:


The Growing T
-What's the rule?
The T grows by add 1 □.
-What's the magic number?
4
Look!
step | number of □ | equation
1 | 5 | 1+4=5
2 | 6 | 2+4=6
3 | 7 | 3+4=7
4 | 8 | 4+4=8
5 | 9 | 5+4=9
6 | 10 | 6+4=10

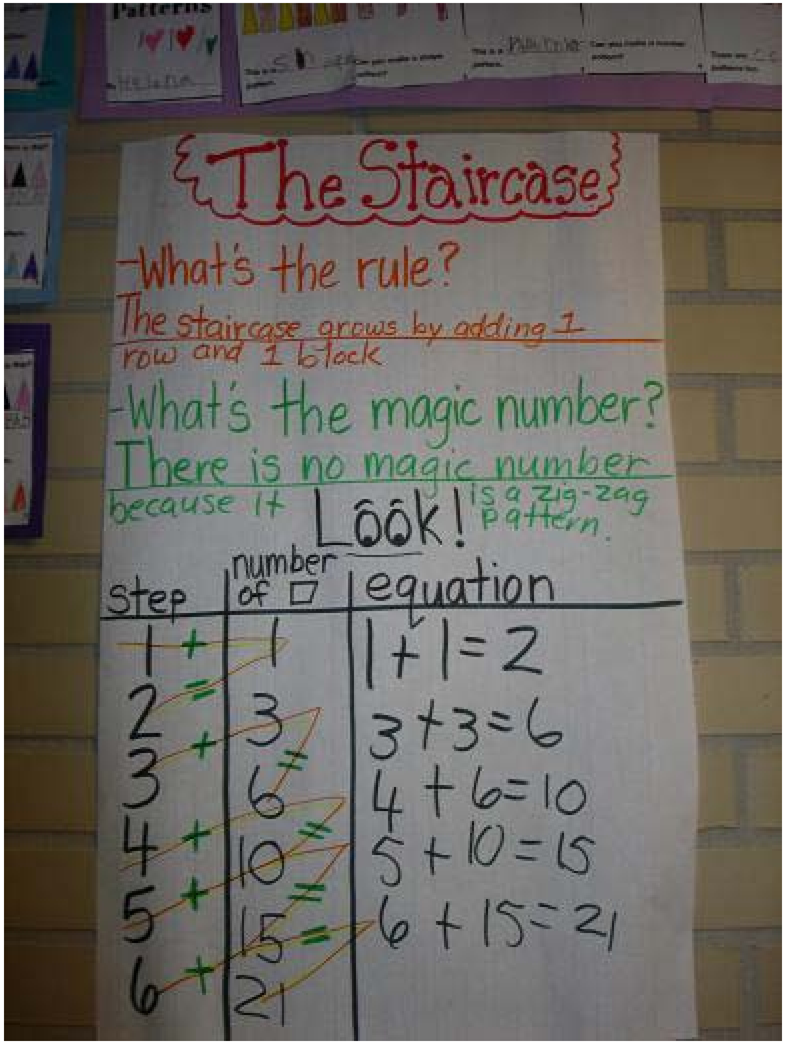

The Growing T
-What's the rule?
The T grows by add 1 □.
-What's the magic number?
4
Look!
step | number of □ | equation
1 | 5 | 1+4=5
2 | 6 | 2+4=6
3 | 7 | 3+4=7
4 | 8 | 4+4=8
5 | 9 | 5+4=9
6 | 10 | 6+4=10
The Growing T
The Growing T
Names:

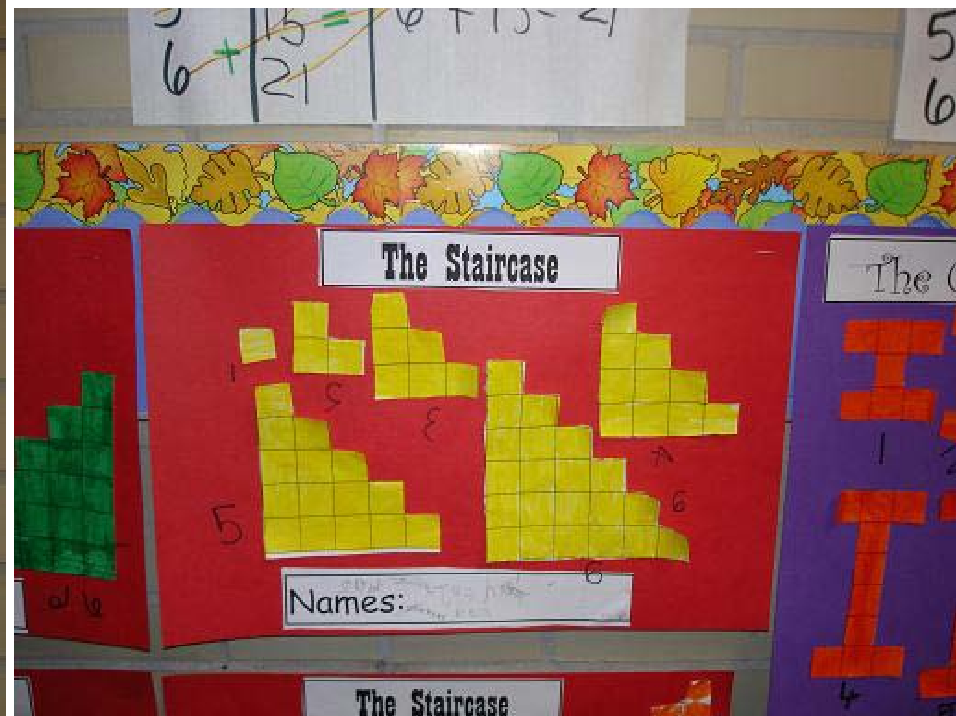


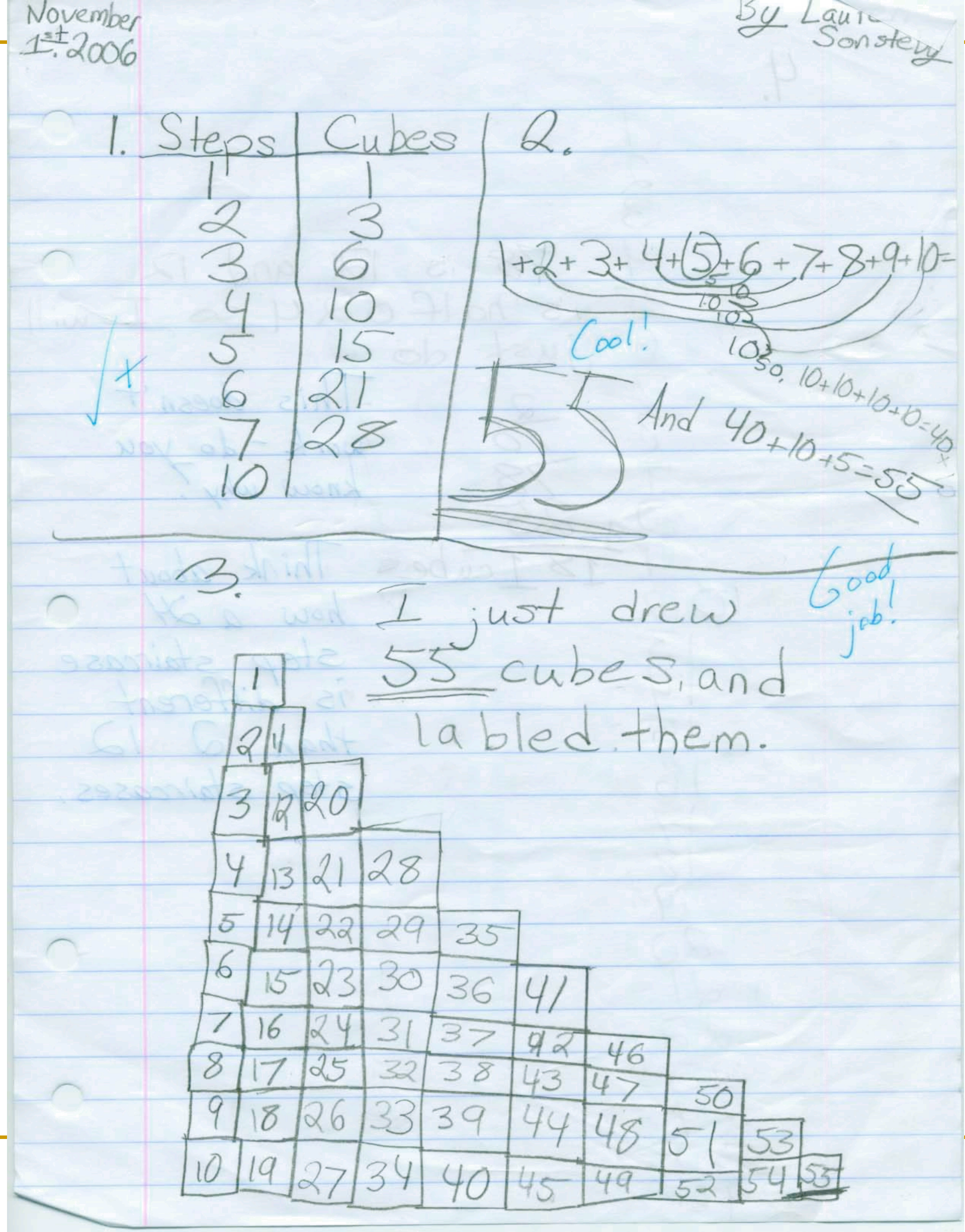
3 9 3 + 6 9
4 10 4 + 6 = 10
5 11 5 + 6 = 11
6 12 6 + 6 = 12
The Growing I
Names:
The Growing I
Names:
The Growing I
The Growing I



The Growing I
-What's the rule?
The I grows by adding 1□
-What's the magic number?
6
Look!
step
number of □
equation
1 7 1 + 6 = 7
2 8 2 + 6 = 8
3 9 3 + 6 9
4 10 4 + 6 = 10
5 11 5 + 6 = 11
6 12 6 + 6 = 12


The Staircase
-What's the rule?
The staircase grows by adding 1 row and 1 block
-What's the magic number?
There is no magic number because it is a zig-zag pattern.
Look!
step
number of □
equation
1+1=2
3+3=6
4+6=10
5+10=15
6+15=21


The Staircase
Names:
The Staircase



November 1st, 2006

By Laur… Sonsteny

1.

Steps	Cubes
1	1
2	3
3	6
4	10
5	15
6	21
7	28
10	

2.

1+2+3+4+(5)+6+7+8+9+10=

10 10 10 10 So, 10+10+10+10=40

And 40+10+5=55

55

Cool!

3. I just drew 55 cubes, and labled them.

Good job!

Lesson 4: How Many Triangles Can You Construct?

Students identify patterns in a geometrical figure (based on triangles) and build a foundation for the understanding of fractals. A **fractal** is "a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole“

Learning Objectives

Students will:

identify patterns in a geometrical figure

build a foundation for the understanding of fractals

make hypothesis and then develop experiments to test them

Materials

How Many Triangles? activity sheet

Let's Work Together Family Page (photocopied on cardstock)

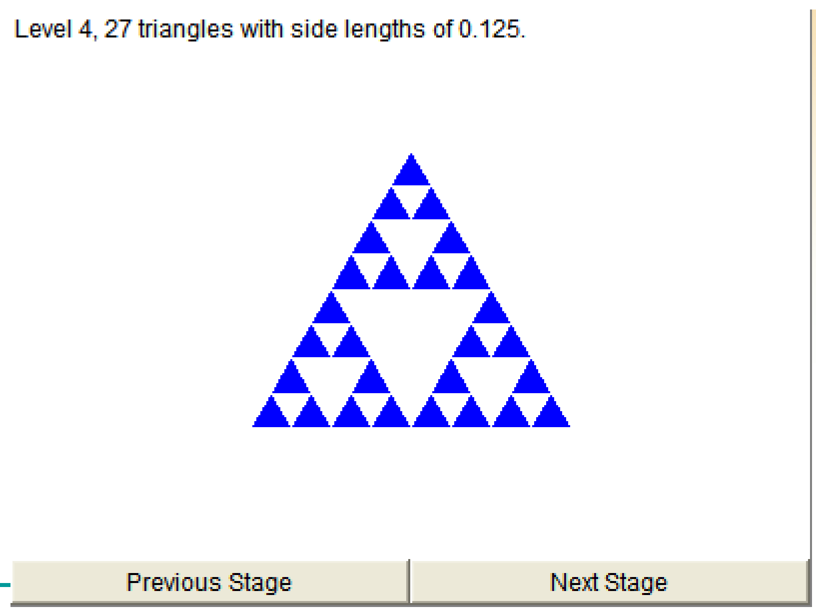
Ruler, pencils, or fine-line markers

Writing paper

Lesson 4: How Many Triangles Can You Construct?

What is Sierpinski's Triangle?

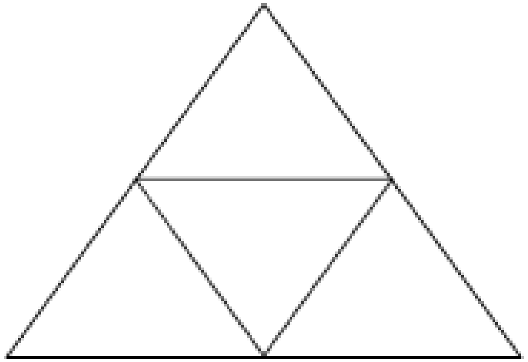
http://www.shodor.org/interactivate/activities/SierpinskiTriangle/



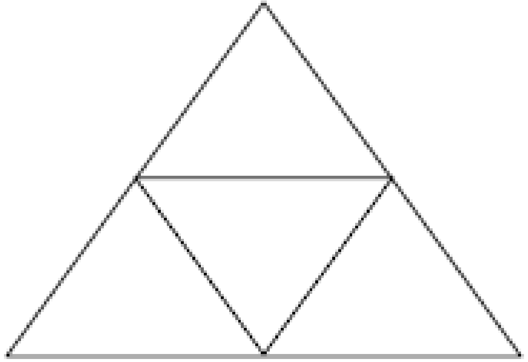
How Many Triangles?

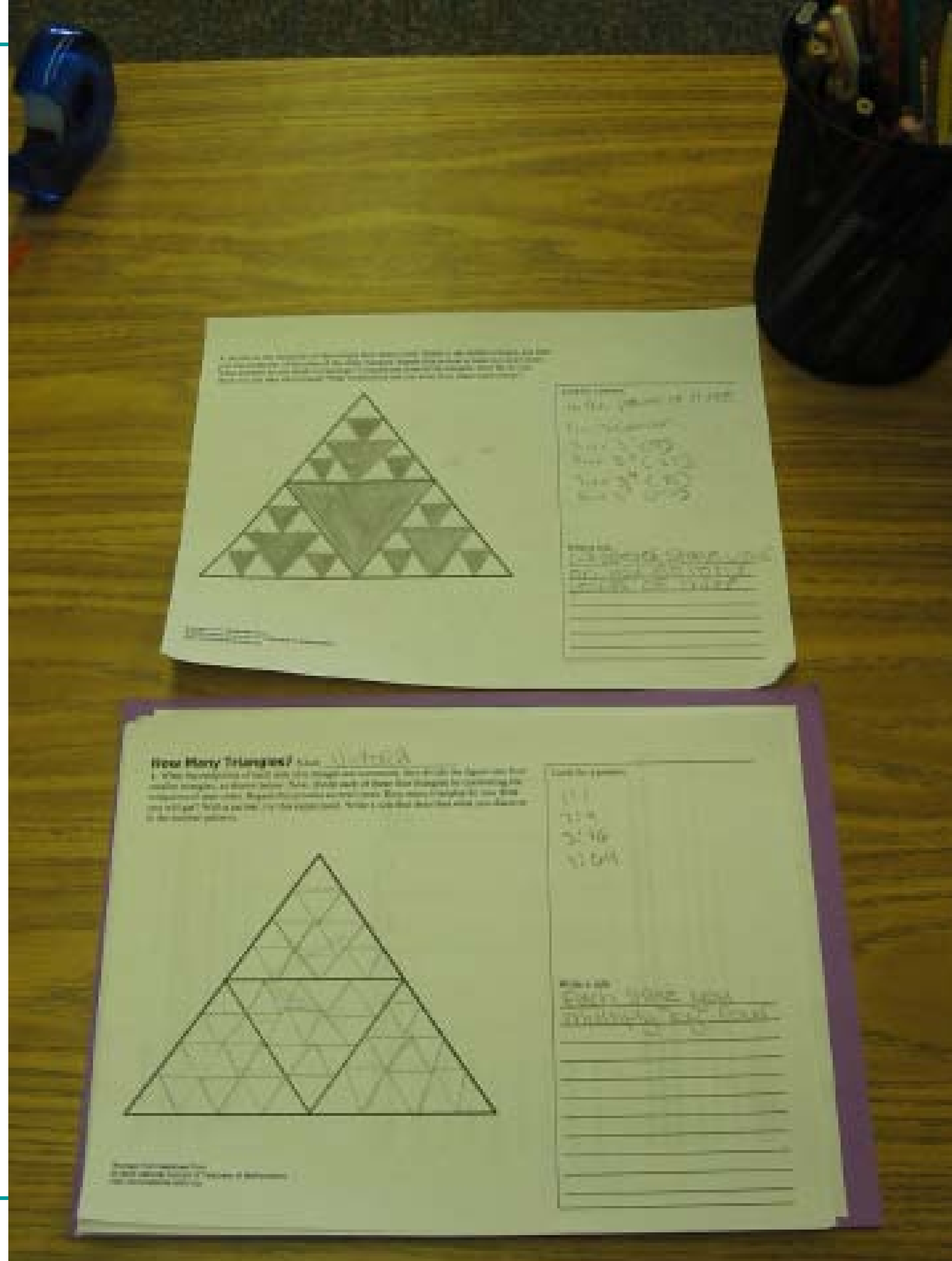
NAME ______________________

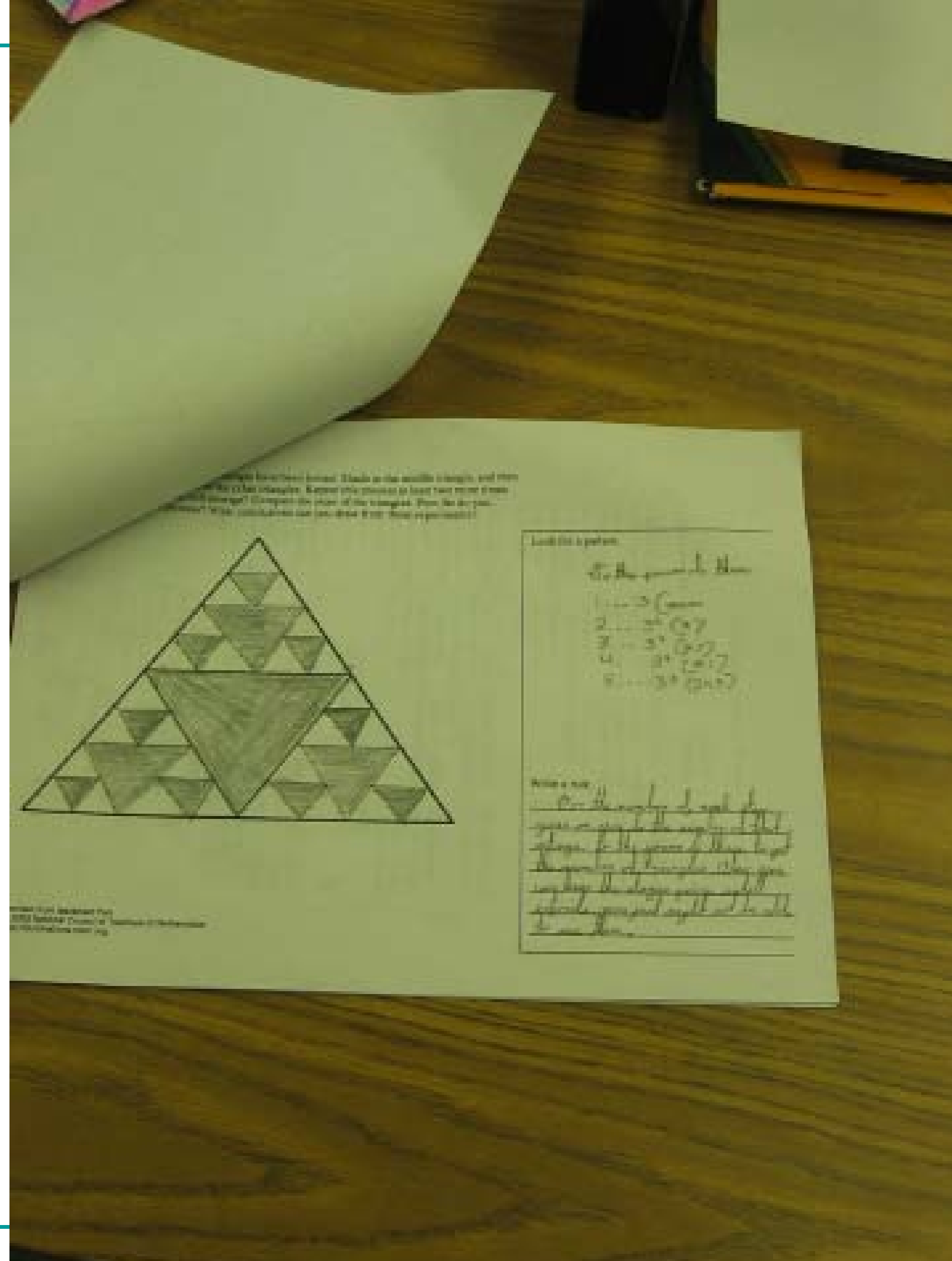
1. When the midpoints of each side of a triangle are connected, they divide the figure into four smaller triangles, as shown below. Now, divide each of these four triangles by connecting the midpoints of their sides. Repeat this process several times. How many triangles do you think you will get? With a partner, try this experiment. Write a rule that describes what you discover in the number patterns.

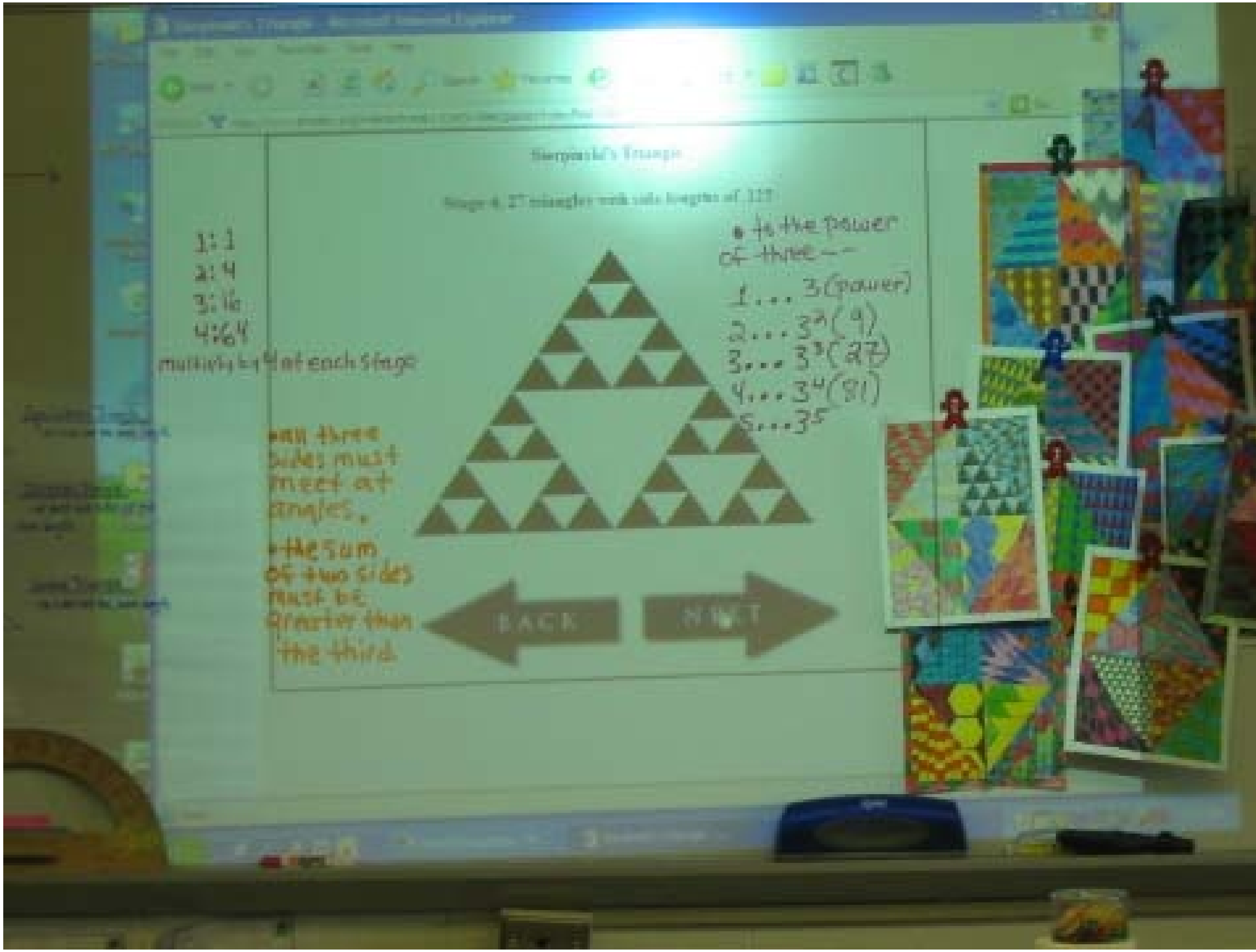


2. As above, the midpoints of the triangle have been joined. Shade in the middle triangle, and then join the midpoints of the sides of the other triangles. Repeat this process at least two more times. What patterns do you think will emerge? Compare the sizes of the triangles. How far do you think you can take this process? What conclusions can you draw from these experiments?



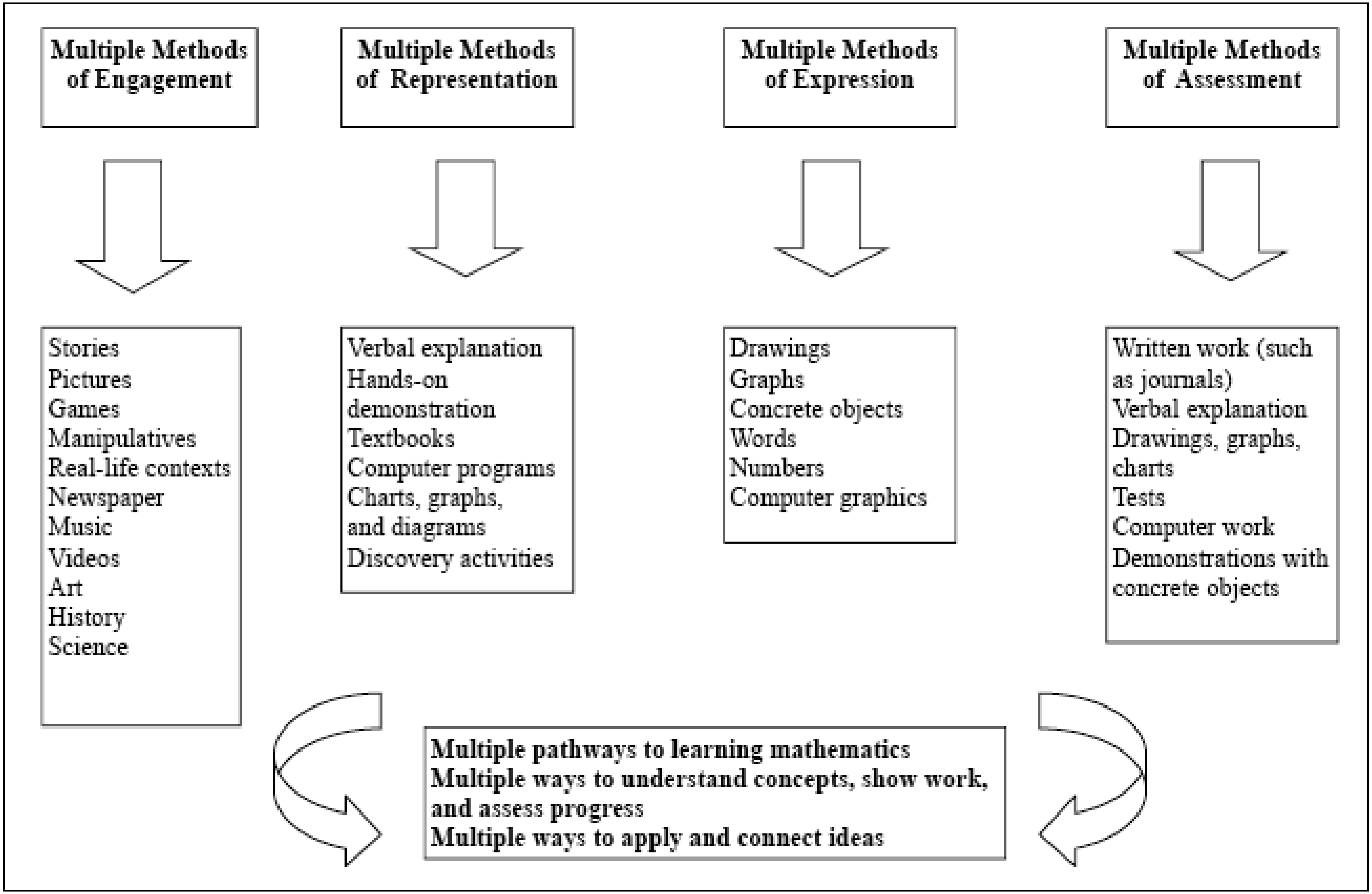

How Many Triangles?

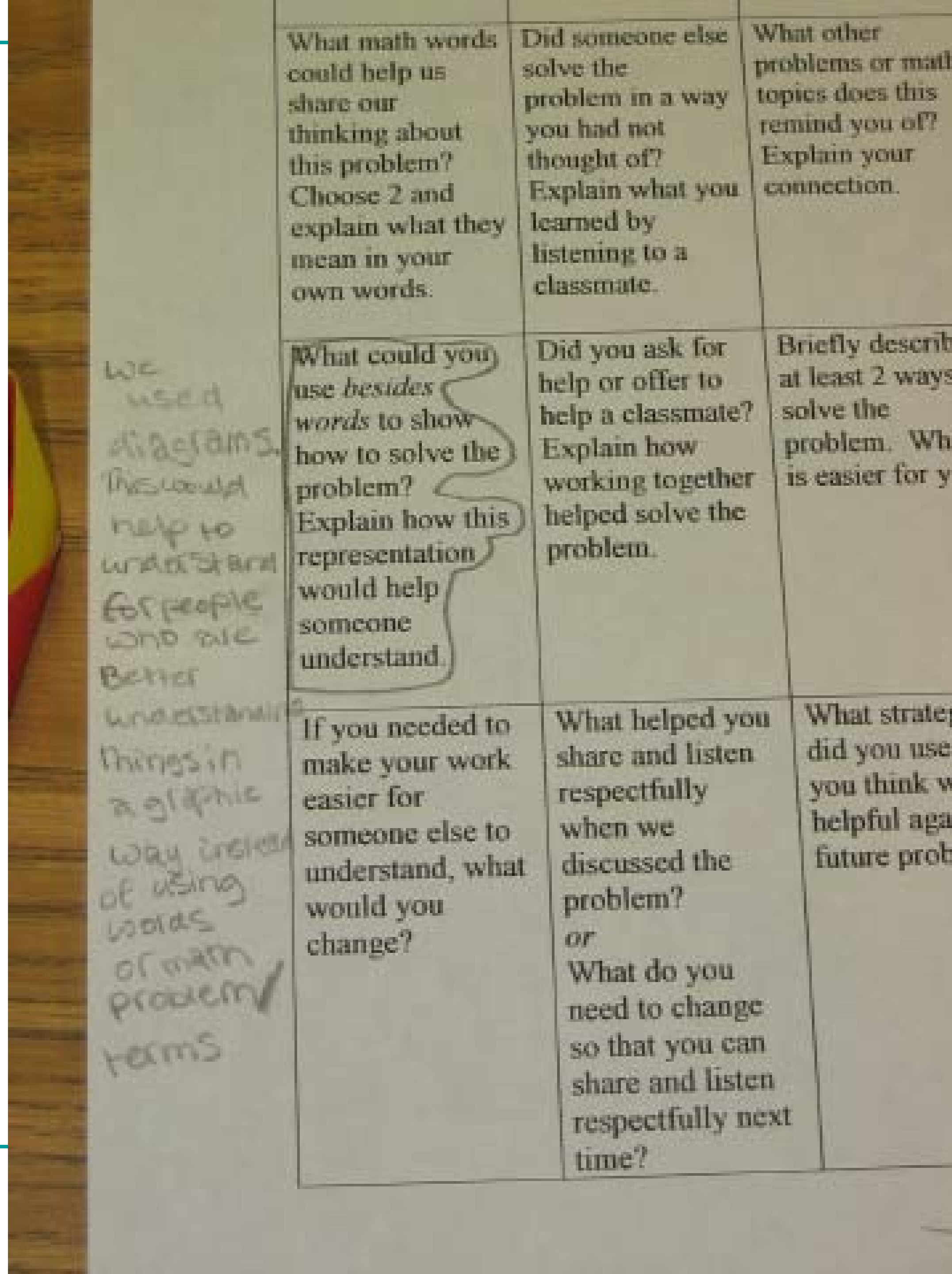




1:1
2:4
3:16
4:64
multiply by 4 at each stage
all three sides must meet at angles.
the sum of two sides must be greater than the third
to the power of three
1... 3(power)
2... 3² (9)
3... 3³ (27)
4... 3⁴ (81)
5... 3⁵
BACK
NEXT

Universal Design in Math Instruction





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What could you use *besides words* to show how to solve the problem? Explain how this representation would help someone understand.	Did you ask for help or offer to help a classmate? Explain how working together helped solve the problem.	Briefly describ at least 2 ways solve the problem. Whi is easier for yo
If you needed to make your work easier for someone else to understand, what would you change?	What helped you share and listen respectfully when we discussed the problem? *or* What do you need to change so that you can share and listen respectfully next time?	What strateg did you use t you think w helpful agai future probl

We used diagrams. This would help to understand for people who are Better understanding things in a graphic way instead of using words or math problem/ terms

Reflection on Problem solving
From Brooke's class…

A} What did you do if you got "stuck" or felt frustrated?

--We asked for help and we tried to look at things in a different way. Sam & Laura

B} What strategies did you use that you think will be helpful again for future problems?

--I think a strategy I will use again is the making of visual things that help me through the problem. Mikey

--I used division to help me see if I could form a triangle. Grant

Reflection on Problem solving
From Brooke's class…

C} Did you ask for help or offer to help a classmate? Explain how working together solved the problem.

--I asked for help and offered help. I think working in groups is easier because two people can do more than one. Jonathan

--We told each other what we could do to make it easier and explained our ideas. Maura

--I asked my partner and it was good because then she would ask me something and we worked together on different parts. Christina

D} What could you use *besides words* to show how to solve the problem? Explain how this representation would help someone understand.

--I believe diagrams trigger peoples' minds so they understand and visualize the problem better. I don't know about other people, but it helps me. Gaven

--We used diagrams. This could help to understand for people who are better understanding things in a graphic way instead of using words or main problem solving terms. Victoria

--If you find the rule and the pattern, you can better see how a problem works. Nick

Reflections from Gwen's Class…

Flexible Thinking

"Using the formula to predict if the sides would make a triangle helped me a lot. It is a good strategy for the future." Sam

"This problem reminded me of the shapes that we made with the straws and twist-ties." Danielle

"I like trial-and-error because you start with a big guess and narrow it down." Griffin

"A strategy that will help me in the future would be the rule that we found out today." Emma

"A strategy that I would use again after this problem would be guessing. I think this because many problems involve estimating. I'm guessing more and doing it better." Alex

"I think that doing the number sentences will help me in the future." Molly

"This reminds me of when we tried to find perimeter in the beginning of the year. When we first did this, we could barely multiply and divide." Liam

Reflections from Gwen's Class…

Persistence

"I feel much more confident in math, because this problem showed me different problems, strategies, and persistence. The persistence helped me because I put my mind to it." Alex

"What helped me try my best was when Michael didn't understand something and made me know I had to try harder to explain it better." Liam

"I felt more confident about math after trying this problem because I proved to myself that if I am persistent, then I can accomplish things in math that I set my mind to." Lauren

"I feel a lot more confident about math after those problems because I know what it feels like to be persistent, and I like it! So I'm going to keep going for that feeling. Emily

"What helped me to do my best was the hard questions. The more confusing it was, the more I liked it to try my best." Liam

Final Thoughts…

Growth mindset:

“I just keep going like a snow plow stuck in the road. I didn’t wait for the spring to come. I kept going.” Griffin