Developing Persistent Flexible Problem Solvers

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Lesson Study Cycle

1. SET OVERARCHING GOALS
   Consider long term goals for student learning and development
   Study curriculum and standards

2. PLAN
   Select & revise research lesson; design artifact
   Anticipate student responses
   Plan data collection and lesson

3. RESEARCH LESSON
   Conduct research lesson
   Team observes the lesson and collect data on student learning

4. REFLECT & REVISE
   Share data
   What was learned about students learning, lesson design, this content?
   What are implications for this lesson and instruction more broadly?
   Revise and repeat.
“What kind of a problem solver would you like to have in your class?”
Flexibility

- Good problem solvers are flexible and resourceful.

They have many ways to think about problems—alternative approaches if they get stuck, ways of making progress when they hit roadblocks, of being efficient with (and making use of) what they know.

They also have a certain kind of mathematical disposition—a willingness to pit themselves against difficult mathematical challenges under the assumption that they will be able to make progress on them, and the tenacity to keep at the task when others have given up.
Beliefs about learning and intelligence also influence mathematics performance. When faced with challenging problems, children who believe that intelligence is in large part created by their efforts to learn tend to do better than children who believe that intelligence is a fixed quality that cannot be changed (Dweck, 1999).

From the National Math Panel report 2008
Intelligence = _____ % Ability _____ % Effort

Growth Mindset
Fixed Mindset

Mindset: The New Psychology of Success (Dweck, 2006)
Fixed Mindset

- Avoids challenges
- Gives up easily
- Sees effort as fruitless or worse
- Ignores useful negative feedback
- Feels threatened by the success of others
Growth Mindset

- Embraces challenges
- Persists in the face of setbacks
- Sees effort as the path to mastery
- Learns from criticism
- Finds lessons and inspiration in the success of others
Self-efficacy

- Student beliefs about the causes of their success and failure have been repeatedly linked to their engaging and persisting in learning activities. Self-efficacy has emerged as a significant correlate of academic outcomes.

- Students can attribute their successes and failures to ability (e.g., I’m just good/bad) at mathematics, effort (e.g., I worked/did-not-work hard enough), luck, or powerful people (e.g., the teacher loves/hates me). These attributions influence students’ subsequent engagement in learning.
<table>
<thead>
<tr>
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<th>Flexible Thinking</th>
<th>Persistence</th>
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Patterns that Grow!

The Growing T

Tommy is building the letter T with cubes. This is what he started with.

Tommy continues to add another cube to his letter T. The next stage looks like this:

He wanted to add another block so that is letter T is growing longer. The third stage looks like this:
Patterns that Grow

1. What would Tommy’s T look like if he continues the pattern to the fourth stage?

2. What would Tommy’s T look like if he continues the pattern to the fifth stage?

3. What would Tommy’s T look like if he continues the pattern to the tenth stage?

4. What was the pattern you noticed? Explain

5. Create a formula or a rule that you noticed from the pattern.
Sara is building a skyscraper with her Legos. She needs to make a staircase to get to the top.

When it was one step high it looked like this:

When it was two steps high it looked like this:

When it was three steps high it looked like this:

How many Legos did she use when the staircase was:

1 step high? ____________

2 steps high? ____________

3 steps high? ____________

4 steps high? ____________

5 steps high? ____________
What do you notice about the number of Legos used as the staircase gets higher?

____________________________________________________________________________________

How many Legos will Sara need to make her staircase 10 steps high? (Use the space below to show how you solved this problem. Explain using pictures, numbers, or words).

Number of Legos: ______________

Show work below:
**The Growing T**

- What's the rule?
  The T grows by add 1.
- What's the magic number?

**Look!**

<table>
<thead>
<tr>
<th>step</th>
<th>number of T</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1 + 4 = 5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2 + 4 = 6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3 + 4 = 7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4 + 4 = 8</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>5 + 4 = 9</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>6 + 4 = 10</td>
</tr>
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</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3 + 4 = 7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4 + 4 = 8</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>5 + 4 = 9</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>6 + 4 = 10</td>
</tr>
</tbody>
</table>
The Growing I

What's the rule?
The I grows by adding 2.

What's the magic number?
6

Look!

<table>
<thead>
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<th>number of</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1 + 6 = 7</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2 + 6 = 8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3 + 6 = 9</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>4 + 6 = 10</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>5 + 6 = 11</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>6 + 6 = 12</td>
</tr>
</tbody>
</table>
The Staircase

What's the rule?
The staircase grows by adding 1 row and 1 block.

What's the magic number?
There is no magic number because it looks like a zig-zag pattern.

Number of blocks

<table>
<thead>
<tr>
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<th>number of blocks</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1 + 1 = 2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3 + 3 = 6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4 + 6 = 10</td>
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<td>4</td>
<td>10</td>
<td>5 + 10 = 15</td>
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<tr>
<td>5</td>
<td>15</td>
<td>6 + 15 = 21</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td></td>
</tr>
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### Problem 1

<table>
<thead>
<tr>
<th>Steps</th>
<th>Cubes</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
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<tr>
<td>3</td>
<td>6</td>
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<td>15</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td></td>
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\[+2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55\]

\[\text{Cool!} \]

And \(40 + 10 + 5 = 55\)

### Problem 3

I just drew 55 cubes, and labeled them.

---

In the diagram below, the numbers represent the count of cubes at each level. The diagram is a 10x10 grid, with each number indicating the total number of cubes up to that level.
Lesson 4: How Many Triangles Can You Construct?

Students identify patterns in a geometrical figure (based on triangles) and build a foundation for the understanding of fractals. A fractal is "a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole."

Learning Objectives
Students will:
identify patterns in a geometrical figure
build a foundation for the understanding of fractals
make hypothesis and then develop experiments to test them

Materials
- How Many Triangles? activity sheet
- Let's Work Together Family Page (photocopied on cardstock)
- Ruler, pencils, or fine-line markers
- Writing paper
Lesson 4: How Many Triangles Can You Construct?

What is Sierpinski's Triangle?
http://www.shodor.org/interactivate/activities/SierpinskiTriangle/
How Many Triangles?

1. When the midpoints of each side of a triangle are connected, they divide the figure into four smaller triangles, as shown below. Now, divide each of these four triangles by connecting the midpoints of their sides. Repeat this process several times. How many triangles do you think you will get? With a partner, try this experiment. Write a rule that describes what you discover in the number patterns.

![Diagram of a triangle divided into smaller triangles]

2. As above, the midpoints of the triangle have been joined. Shade in the middle triangle, and then join the midpoints of the sides of the other triangles. Repeat this process at least two more times. What patterns do you think will emerge? Compare the sizes of the triangles. How far do you think you can take this process? What conclusions can you draw from these experiments?

![Diagram showing additional divisions of the triangle]
Look for a pattern.

1. 2^1 3
2. 2^2 3
3. 2^3 3
4. 2^n 3^n

Write a rule for generating a row of the triangle.
Sierpinski's Triangle

Stage 4, 27 triangles with side lengths of 1/3:

1: 1
2: 4
3: 16
4: 64

Multiples at each stage:

- The sum of two sides must be greater than the third.
- Three sides meet at each stage.

\( a \text{ to the power of three} = \)

1: \( 3^1 \) (power)
2: \( 3^2 \) (9)
3: \( 3^3 \) (27)
4: \( 3^4 \) (81)
5: \( 3^5 \)
Universal Design in Math Instruction

Multiple Methods of Engagement
- Stories
- Pictures
- Games
- Manipulatives
- Real-life contexts
- Newspaper
- Music
- Videos
- Art
- History
- Science

Multiple Methods of Representation
- Verbal explanation
- Hands-on demonstration
- Textbooks
- Computer programs
- Charts, graphs, and diagrams
- Discovery activities

Multiple Methods of Expression
- Drawings
- Graphs
- Concrete objects
- Words
- Numbers
- Computer graphics

Multiple Methods of Assessment
- Written work (such as journals)
- Verbal explanation
- Drawings, graphs, charts
- Tests
- Computer work
- Demonstrations with concrete objects

Multiple pathways to learning mathematics
- Multiple ways to understand concepts, show work, and assess progress
- Multiple ways to apply and connect ideas
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We used diagrams. This could help to understand for people who are better understanding things in a graphic way instead of using words or math problem terms.
Reflection on Problem solving

From Brooke’s class…

A} What did you do if you got “stuck” or felt frustrated?

--We asked for help and we tried to look at things in a different way. Sam & Laura

B} What strategies did you use that you think will be helpful again for future problems?

--I think a strategy I will use again is the making of visual things that help me through the problem. Mikey

--I used division to help me see if I could form a triangle. Grant
Reflection on Problem solving
From Brooke’s class…

C) Did you ask for help or offer to help a classmate? Explain how working together solved the problem.

--I asked for help and offered help. I think working in groups is easier because two people can do more than one. Jonathan
--We told each other what we could do to make it easier and explained our ideas. Maura
--I asked my partner and it was good because then she would ask me something and we worked together on different parts. Christina

D) What could you use *besides words* to show how to solve the problem? Explain how this representation would help someone understand.

--I believe diagrams trigger peoples’ minds so they understand and visualize the problem better. I don’t know about other people, but it helps me. Gaven
--We used diagrams. This could help to understand for people who are better understanding things in a graphic way instead of using words or main problem solving terms. Victoria
--If you find the rule and the pattern, you can better see how a problem works. Nick
Reflections from Gwen’s Class…

Flexible Thinking
“Using the formula to predict if the sides would make a triangle helped me a lot. It is a good strategy for the future.” Sam

“This problem reminded me of the shapes that we made with the straws and twist-ties.” Danielle

“I like trial-and-error because you start with a big guess and narrow it down.” Griffin

“A strategy that will help me in the future would be the rule that we found out today.” Emma

“A strategy that I would use again after this problem would be guessing. I think this because many problems involve estimating. I’m guessing more and doing it better.” Alex

“I think that doing the number sentences will help me in the future.” Molly

“This reminds me of when we tried to find perimeter in the beginning of the year. When we first did this, we could barely multiply and divide.” Liam
Reflections from Gwen’s Class…

**Persistence**

“I feel much more confident in math, because this problem showed me different problems, strategies, and persistence. The persistence helped me because I put my mind to it.” Alex

“What helped me try my best was when Michael didn’t understand something and made me know I had to try harder to explain it better.” Liam

“I felt more confident about math after trying this problem because I proved to myself that if I am persistent, then I can accomplish things in math that I set my mind to.” Lauren

“I feel a lot more confident about math after those problems because I know what it feels like to be persistent, and I like it! So I’m going to keep going for that feeling. Emily

“What helped me to do my best was the hard questions. The more confusing it was, the more I liked it to try my best.” Liam
Final Thoughts…

Growth mindset:

“I just keep going like a snow plow stuck in the road. I didn’t wait for the spring to come. I kept going.” Griffin