A qualitative energy-based unified representation for buildings

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Abstract. This paper presents the development of energy-based unified representations for buildings called Archi Bond Graphs and then specialises them as qualitative Archi Bond Graphs that combine graphical representations and qualitative equations. They can be applied to simulations of people behaviour and people-energy behaviour in space-people systems, building energy flows and building energy variations in different building energy systems, and energy interactions between these building subsystems. The applicability of qualitative Archi Bond Graphs is demonstrated through a building simulation for the dynamic energy interactions between the space-people system and building energy systems, including lighting and hydraulic systems, in a town-house design.

Keywords: bond graphs, building analyses, qualitative reasoning, representation, system dynamics

1. Introduction

Representations enable designers to establish a framework to describe designs and also provide a foundation for their evaluation. Representations for building design and building analysis can be in different forms. Currently, there is a lack of a unified representation that can be used for representing and analysing different building subsystems simultaneously, Fig. 1. Most representations cannot be uniformly applied in the conceptual, intermediate, and final stages of building design. Current commercial representations such as CAD systems are generally used in the final stage of building design. Representations for use during the conceptual stage of building design are still immature. There are few representations that can represent both qualitative and quantitative aspects of buildings homogeneously. There are few representations that can represent both static aspects, e.g. spatial arrangements, and dynamic aspects, e.g. people movement and building energy movements within buildings.

A unified representation should be applicable to different building subsystems and be applicable at
the conceptual, intermediate, and final design stages. It should also have the capacity to represent both static and dynamic aspects of buildings. This unified representation should provide designers a foundation for effective and efficient communication, understanding, and evaluation of the effects of interactions between different building subsystems.

Bond graphs introduced by Paynter [7,14,21,22] are energy-based representations. They are a class of graphical languages and systematic representations. Bond graphs have the potential to allow for the mixing of qualitative and quantitative features in a single representation. As a modelling tool, bond graphs can be used in the conceptual design stage as well as later in the design. Qualitative reasoning [15,26] is a powerful model-based reasoning method which allows for reasoning with non-numeric, incomplete and weak numerical information. Qualitative reasoning can be employed to build a deep-level knowledge model to represent the relationship between system structure and behaviour [25-27]. The development of a unified representation for buildings described in this paper is based on bond graphs. It then uses qualitative reasoning to specialise this unified representation.

Section 2 very briefly introduces bond graphs and qualitative reasoning. Section 3 presents the development of a unified representation for buildings based on bond graphs. Section 4 takes this bond graph representation and extends it to include qualitative reasoning. Section 5 shows how such a qualitative architectural bond graph can be used for both simulation and inference of the behaviour of people and systems in a building. This is followed, in Section 6, by a detailed example of an application of this approach, with results showing the qualitative behaviour of the building.

2. Bond graphs and qualitative reasoning for architectural analysis

2.1 Bond graphs

The bond graph is a tool that provides a unified approach to the modelling and analysis of dynamic systems, especially for hybrid multi-domain systems including mechanical, electrical, pneumatic, and hydraulic systems [3,5,14]. Bond graphs combine graphical representations and mathematical equations and consist of variables, elements and constitutive relations. Table 1 shows bond graph variables of effort (e), flow (f), momentum (p), displacement (q), power (P), and energy (E) for mechanics, electronics, hydraulics, and economics. Bond graphs consist of nine types of elements. They are source of effort (Se), source of flow (Sf), inductor (I), capacitor (C), resistor (R), transformer (TF), gyrator (GY), 0-junction
and 1-junction. Transformer (TF) does not create, store or dissipate energy. It conserves power and transmits the factors of power with power scaling as defined by the transformer modulus. Gyrator (GY) is used in most of the cases where power from one energy domain is transferred to another. 0-junction and 1-junction stand for parallel junction and serial junction respectively. 0-junction and 1-junction also serve to interconnect elements into subsystems or systems. Bond graphs elements can be divided into three categories: one-port elements, two-port elements, and three or multi-port elements. Table 2 shows elements of bond graphs with their symbols and examples. Table 3 shows I, C, and R-elements of bond graphs of mechanics, electronics, hydraulics, and economics. Fig. 2(a) is an example of the graph of an electrical system; Fig. 2(b) shows the bond graph of the same electrical system.

Bond graphs can be used to express the common energy transfer or transformation underlying several domains with a uniform notation. The characteristics of bond graphs include being able to:

- reveal the relationships among a system, subsystems, and elements;
- obtain important qualitative information as a conceptual model;
- represent the topology of subsystems or elements of different domains, as well as represent the relationships of their products;
- represent dynamic systems; and
- be flexible and extendable.

Applications of bond graphs in architecture appear to have been limited to greenhouse dynamic modelling [2] and HVAC systems design and simulation [29]. These researches demonstrate some of the concepts of applying bond graphs in the domain of architecture. However, both of them focus on limited specific subsystems of buildings. People and spaces which are often discussed in architectural design and building analyses are not considered. People moving within a building can move in two directions, including moving into and out of rooms in the building. However, bond graphs have been primarily applied to situations with uni-directional flows.

2.2 Qualitative reasoning for architectural analysis

Qualitative reasoning

Qualitative reasoning is concerned with qualitative values for variables and relationships. It can be employed to build a deep-level knowledge model to represent the relationship between system structure
and behaviour without the need for direct and explicit knowledge of the underlying laws of physics [1,25-27]. Research in qualitative reasoning has addressed model building and model simulation problems. The former selects an appropriate model or combination of models to answer a particular question. The latter simulates the model to make explicit some facts about the world that are implicit in that description of the world [15].

An effective qualitative model should possess the following capabilities [4]:

- describe explicitly the locations of system components and their interconnections;
- describe simply the relations between system input and output;
- be applicable across domains to represent complex dynamic systems with a uniform format and should be systematically built by using a formal modelling language;
- allow the integration of qualitative representations and quantitative information; and
- overcome the difficulty of higher-order qualitative derivatives.

**Graph representation and qualitative reasoning for architecture**

A graph is a set of vertices connected by edges. Depending on the application, edges may have directions or not. Vertices and edges may be assigned weights using numbers [28]. Graphs in architectural design have been used to represent a variety of spatial structures and the adjacencies between rooms in a plan [20]. For a plan, each room in an arrangement is represented by a vertex.

Graphs can be interpreted as a qualitative representation of a model [26]. Graph representation and qualitative reasoning can be applied in the conceptual building design stage when there is incomplete or poor numerical information. Current research into qualitative reasoning for architectural analyses mainly focuses on the static aspects of 2D shapes, 2D spaces, and 3D objects for their similarity and complexity [8-10].

3. **The development of a unified representation for buildings**
   3.1 **From regular bond graphs to bond graphs for multiple domains and Archi Bond Graphs**

The development of an energy-based unified representation for buildings is based on bond graphs [7,14,19,21,22,25], which are called regular bond graphs (RBGs) in this paper. These are generalised for multiple domains (MBGs). MBGs possess more general definitions for the variables and elements. They
have the capacity to integrate and to be applied to multiple domains based on the energy transfer and transformation concept. MBGs are further specialised to the domain of architecture to develop Archi Bond Graphs (ABGs).

The ABG representation is a qualitative and quantitative energy-based unified representation for buildings. ABGs are applied to three main aspects of a building: as a collection of objects, as a collection of people and their goods, and as a container and transformer of processes. Fig. 3 shows the development of a unified representation for building analysis from RBGs to MBGs and ABGs.

### 3.2 Bond graphs for multiple domains (MBGs)

In the development of bond graphs for multiples domains (MBGs), in order to find a more general energy definition which can be applied to different energy domains, energy (E) and displacement (q) are the fundamental variables and defined in a broad sense. Energy has the ability or power to do or to achieve something. For instance, in the domain of economics, money represents energy, since money is the source of power for buying and has the ability to put effort into economic activities. Displacement is the change caused by energy. Change in different domains could also be different things, such as displacement in mechanics, charge in electronics, volume in hydraulics, and order in economics.

Based on the above definitions and complying with the constitutive relations, the other MBG variables of effort (e), flow (f), momentum (p), and power (P) are defined as follows:

- **Effort**, e, is the amount of energy that is needed for a unit of change and is a unit energy. It is equal to $\frac{\text{energy}}{\text{change}}$.

- **Flow**, f, is the number of changes that happen in a unit of time. It is equal to $\frac{\text{change}}{\text{time}}$.

- **Momentum**, p, is the amount of energy that is needed in a period of time for a unit of change. It is equal to $\frac{\text{energy} \times \text{time}}{\text{change}}$ and is also equal to the time integral of effort, Eq (1):

$$p = \frac{\text{energy} \times \text{time}}{\text{change}} = \int e \, dt$$

(1)

- **Displacement**, q, has been defined as $\text{change}$. It is equal to the time integral of flow, Eq (2):

$$q = \int f \, dt$$

(2)

- **Power**, P, is equal to effort times flow and is also equal to the time differential of energy, Eq (3):
Further, I, C, and R-elements of MBGs are developed as follows, which also comply with the constitutive relations:

- I-element is a unit of flow that can cause or store the amount of energy in a period of time. It is equal to momentum over flow, Eq (4);

\[ I = \frac{P}{f} = \frac{\text{energy} \times \text{time}}{\text{change}} = \frac{\text{energy} \times \text{time}^3}{\text{change}^2} \]  

(4)

- C-element is a unit of effort that can cause or store the amount of changes. It is equal to change over effort, Eq (5).

\[ C = \frac{q}{e} = \frac{\text{change}}{\text{energy}} \times \frac{\text{change}^2}{\text{energy}} \]  

(5)

- R-element is a unit energy needed or dissipated for a unit of flow. It is equal to effort over flow, Eq (6).

\[ R = \frac{e}{f} = \frac{\text{energy}}{\text{change}} \times \frac{\text{energy} \times \text{time}}{\text{change}^2} \]  

(6)

In bond graphs there are two kinds of junctions: 0-junctions and 1-junctions:

- 0-junction is similar to a parallel junction. For a 0-junction, efforts on the bonds connected to this 0-junction are all equal and the sum of all connected flows is zero, Eq (7)

\[ \sum f = \sum \frac{\text{change}}{\text{time}} = 0 \]  

(7)

- 1-junction is similar to a serial junction. For a 1-junction, flows on the bonds connected to this 1-junction are all equal and the sum of all connected efforts is zero, Eq (8)

\[ \sum e = \sum \frac{\text{energy}}{\text{change}} = 0 \]  

(8)

Table 4 shows the variables, elements and junctions of MBGs with their mathematical relationships. The MBG variables and MBG elements can be applied to graphical representations and mathematical equations for simulations of energy transfer and transformation in multiple domains.

3.3 Archi Bond Graphs (ABGs)
Within the framework of MBGs, Archi Bond Graphs, called ABGs, are developed for the domain of architecture. ABGs combine graphical representations and mathematical equations and consist of ABG variables, ABG elements, constitutive relations, and ABG bicausal bonds. They are applicable to space-people systems, space-goods systems, and building energy systems to represent and simulate the static and dynamic aspects of buildings, i.e. the structures of spatial arrangements and building energy arrangements as well as the behaviours of people, goods, and building energy within these structures. A spatial arrangement with people behaviours is called a space-people system, a spatial arrangement with goods behaviours is called a space-goods system, and a building energy arrangement with building energy movements is called a building energy system.

In the development of ABGs, new definitions of ABG variables are proposed and some new ABG elements are developed. Constitutive relations constructing the relationships between variables and elements are represented by mathematical equations. ABG bicausal bonds have the capacity to represent two-directional movements of people, goods, and building energy moving into and out of a space or device within a building as well as the resulting unit energy variations simultaneously.

**ABG variables**

In applying ABGs to the space-people system and the building energy system, for different building energy systems, e.g. lighting systems and hydraulic systems, the definitions and units of variables and elements are very similar to those in RBGs for systems of electricity, hydraulics, and HVAC. For the space-people system, ABG variables of energy (E) and displacement (q) are defined as follows [11,12]:

- Energy, E: *people-energy* represents the capability or power of people to do or to achieve something in a building, such as moving from one space to another space or eating food in a dining room. By doing so, they may consume different building energy, for instance, energy of electrical system, hydraulic system and HVAC. *People-energy* (E) is caused by people-energy-flow (P) varied within a building during a period of time (t), \( E = P \times t = e \times f \times t \). It is the sum of variations of all sub-amounts of people-energy caused by all sub-groups of people-flow moving within a building during a period of time. Initially, the value of people-energy (E) in a building is zero, when there is no person in the building. Therefore, the values of unit people-energy (e), people-change (q) and people-flow (f) are all zero.
• Displacement, q: *people-change* is the net number of people varied within a building during a period of time.

ABG variables of effort (e), flow (f), momentum (p), and power (P) for the space-people system are then defined as follows:

• Effort, e: *unit people-energy* is the average amount of people-energy caused by one person moving within a building. It is equal to people-energy over people-change, Eq (9);

\[
e = \frac{\text{people-energy}}{\text{people-change}} \quad (9)
\]

• Flow, f: *people-flow* is the net number of people varied in a unit of time. It is equal to people-change over time, Eq (10);

\[
f = \frac{\text{people-change}}{\text{time}} \quad (10)
\]

• Momentum, p: *people-impulse* is the average amount of people-energy caused by one person moving within a building during a period of time. It is equal to the time integration of unit people-energy, Eq (11);

\[
p = \int (\text{unit people-energy}) \, dt = \frac{\text{people-energy} \times \text{time}}{\text{people-change}} \quad (11)
\]

• Power, P: *people-energy-flow* is the amount of people-energy caused by all people moving within a building in a unit of time. It is equal to unit people-energy times people-flow as well as people-energy over time, Eq (12).

\[
P = \text{unit people-energy} \times \text{people-flow} = \frac{\text{people-energy}}{\text{time}} \quad (12)
\]

**ABG elements**

ABG elements (L) are categorised into 1-port and multi-port elements represented in Eq (13), where 1-port elements are the source (S), i.e. source of effort (S_e) and source of flow (S_f), inducer (I), capacitor (C), resistor (R), controller (CR), meter (M), and building-component (B), i.e. exterior building-component (B_e) and interior building-component (B_i). Multi-port elements include transformer (TF), transducer (TD), 0-junction and 1-junction, and energy controller (EC). The definitions of these elements are given below:

\[
L = \{ S, I, C, R, CR, M, B, TF, TD, 0, 1, EC \} \quad (13)
\]
\[ S = \{S_e, S_t\} \]
\[ B = \{B_e, B_i\} \]

In ABG 1-port elements, the S-element is an active element, I, C, and R elements are passive elements, and CR, M, and B elements are additive elements. For the space-people system, ABG elements of S, I, C, and R are defined as follows:

- **S-element**: *people-energy source* represents the point where people move from;

- **I-element**: *space-potential* stores people-impulse and facilities people moving in the building. ABG I-element represents passages such as corridors, ramps, steps and stairs within a building;

- **C-element**: *space-capacitor* treats space within a building as a container of people. ABG C-element represents different types and scales of rooms within a building.

- **R-element**: *space-resistor* represents the restrictions which restrict people from moving between spaces within a building. It dissipates energy in the space-people system.

CR, M, and B elements are introduced to ABGs for the space-people system and the building energy systems as controllers, meters, and building-components respectively.

- **CR-element**: *controller* does not dissipate energy but affects energy flow, e.g. slows down or stops energy flow. For instance, it represents doors in the space-people system. In building energy systems, it is a switch in the lighting system and a tap in the hydraulic system.

- **M-element**: *meter* measures the variation(s) of people-energy and/or building energy at specific points in the building.

- **B-element**: *building-component* affects building energy consumption. B-element includes B_e and B_i elements. The former is placed between exterior and interior spaces, e.g. external walls of a building or windows on the façade, and the latter is placed in between different interior spaces, e.g. internal walls or floors.

TF and TD elements of ABGs are energy transformer and transducer respectively. The TF-element conserves power and transmits the factor of power, i.e. effort (e) and flow (f), with power scaling, while the TD-element represents the mechanism where different types of energy are transduced.

ABG 0-junction and 1-junction are a parallel junction and serial junction respectively. For the space-people system they are treated as *space-junctions*, connecting one space to one or more spaces. The differences between these two space-junctions are:
• 0-junction implies that people may progress to spaces different from the space which they came from.

• 1-junction implies that people are not able to progress to other spaces except by returning to the space which they came from.

EC-element, energy controller, introduced in ABGs is applied to energy interaction(s) between different systems as a mechanism which receives the request of building energy needed from the space-people system. It also triggers and controls building energy activated and supplied in/from the building energy system.

Tables 5 and 6 shows ABG variables and ABG elements of the space-people system. For ABGs applied to the space-goods system, goods are a factor of the energy variable (E) which replaces people in ABGs for representing goods flow within buildings. Therefore, the energy (E) of ABGs for the space-goods system is goods-energy and the displacement (q) is goods-change. Other ABG variables and ABG elements of S, I, C, and R for the space-goods system are shown in Table 7.

ABG bicausal bonds

Causality in RBGs establishes the cause and effect relationships between the factors of power. In each bond, the input and output are characterised by the causal stroke. The causal stroke represents the place which flow moves away from and effort moves into. The bicausal bond introduced by Gawthrop [6] is extended to develop bicausal bonds in ABGs. Different from causality in RBGs and Gawthrop’s work, an ABG bicausal bond is attached with two half-arrows and two causal strokes at both ends of the bond, as well as with two pairs of effort and flow. It represents both flow (f), e.g. people-flow, goods-flow, and building energy flow, and the resulting effort (e), e.g. unit people-energy variation, unit goods-energy variation, and effort of building energy variation, in two-directional movements, moving into and out of a space or device simultaneously, Fig. 4. An ABG bicausal bond can be simplified as a bond attached with two half-arrows at both ends of the bond.

A spatial system with a corridor A, rooms B and C, and doors CR1, CR2, and CR3 associated with people flow within it, Fig. 5(a), is used to show ABG bicausal bonds for representing two-directional people movements, moving into and out of a room. People move from the outside through door CR1 to corridor A of the spatial system. Some people move in one direction through door CR2 from the corridor
into room B. Some people move in two directions through door CR3 from the corridor into room C and from room C to the corridor. Fig. 5(b) is the ABG graphical representation of the static spatial structure and the dynamic people movements of this space-people system. Attached with power variables, i.e. variables of flow and effort, every ABG bond with a half-arrow in the dotted line boundary shows one direction of people movement and people-energy variations, and every ABG bond with two half-arrows in the solid grey line boundary shows two directions of people movements and people-energy variations.

**ABG application for building analysis**

The example in Fig. 6 shows the concept of an ABG application for building analysis. It can be in different design stages, i.e. conceptual, intermediate, and final design stages. Different building subsystems, such as a space-people system composed by spatial arrangement with people behaviours and building energy systems A, B, and C composed by different building energy arrangements with building energy movements, are represented in different ABG graphical representations. Different interactions between systems are represented by arcs 1 to 6. Arcs 1 and 2 represent energy interactions between the space-people system and building energy systems B and C. Arcs 3 and 4 represent energy interactions between building energy systems B and C as well as between building energy systems A and C respectively. Arcs 5a, 5b, 6a, and 6b represent energy interactions involving more than two building subsystems. Arcs 5a and 5b are for energy interactions between the space-people system and numerous building energy systems, i.e., one building subsystem interacting with many building subsystems. Arcs 6a and 6b represent energy interactions between the space-people system and building energy system A, which then interacts with another building energy system, building energy system C.

ABGs have the capability to represent system dynamics. Therefore, when the components or relationships are changed in any building subsystem of the space-people system and the building energy system, corresponding ones in the related building subsystems will also be changed. This characteristic will appear in both ABG graphical representations and ABG mathematical equations.

**4. Qualitative approach to Archi Bond Graphs (QABGs)**

Qualitative reasoning is critical for comprehending the problem, formulating a plan for solving the problem, identifying which quantitative laws applied to the problem, and interpreting the results or
outputs of quantitative analysis. It does not reason about a system in terms of the precise values and interrelationships between parameters, but rather reason about these values and interrelationships at a qualitative level [27-29]. A qualitative approach based on both physical laws and expert knowledge and rules is more applicable when mathematical models are difficult to obtain or unavailable [17,18]. The qualitative approach to Archi Bond Graphs, called qualitative Archi Bond Graphs (QABGs) [23,24], draws on qualitative physics and uses discrete symbols to replace numerical parameters for representing and simulating dynamic continuous properties of the building system, such as people behaviours, people-energy variations, building energy movements, and building energy variations. QABGs which combine graphical representations and qualitative equations provide a more general model, applicable to a wide range of different conditions in building analyses. In QABG graphical representations, ABG elements can be categorised into terminals (T) and junctions (J) associated with different element-link relationships. Qualitative equations of QABGs involve qualitative values and qualitative operations.

### 4.1 Elements and element-link-relationships of QABGs

In the QABG graphical representations, elements (QL) and linking bonds represent nodes and arcs. An arc has two pairs of power variables, effort (e) and flow (f), representing two directions of people-flow and the resulting unit people-energy variations moving into and out of a space simultaneously in a space-people system. It can also represent two directions of building energy flow and variations of effort of building energy moving into and out of a device in a building energy system simultaneously. Nodes can be terminals (T) or junctions (J). Terminals include energy source (Tₘ) and energy operators (Tₒ), i.e. inductor (I), capacitor (C), resistor (R), controller (CR), meter (M), and building-component (B). Energy source (Tₘ) is an active 1-port element. In energy operators (Tₒ), passive 1-port elements are inductor (I), capacitor (C), and resistor (R), and additive 1-port elements are controller (CR), meter (M), and building-component (B). Junctions include transformer (TF), transducer (TD), 0-junction (0), 1-junction (1), and energy controller (EC). QABG terminals and junctions are shown in Eq (14) and Table 8.

\[
QL = T \cup J
\]

\[
T = Tₘ \cup Tₒ
\]

\[
Tₘ = \{S\}
\]

\[
Tₒ = \{I, C, R, CR, M, B\}
\]
4.2 Qualitative values, operations and equations of QABGs

Qualitative equations of QABGs associated with qualitative values and qualitative operations provide a mechanism for reasoning about energy transfer and transformation within a system.

Qualitative values and operations of QABGs

The qualitative value of QABGs for variables can be either a landmark value or an open interval between two adjacent landmarks in the qualitative space of the variable. If \( x \) is a variable for QABGs, the sign of its value is \([x]\). Qualitative values of QABGs include \([--], [-], [0], [+], [++]\}. [0] is the boundary between [-] and [+], negative and positive values; [-] and [++] are large negative and large positive values for a variable respectively; and [d] expresses a dependent value which is determined by different conditions of the qualitative operation.

Qualitative values of variables for bond graphs have been discussed elsewhere [13,16,25]. Qualitative values in QABGs apply to:

- power variables, i.e. effort (e) and flow (f), and M and B elements, [-], [0], and [++] represent different abnormal behaviours, while [-] and [+] denote the normal behaviours;
- I, C, R and CR elements, [+] denotes the normal behaviour;
- R and CR elements, [++] denotes element blocked, and [0] denotes element leakage or short circuit.

In the following discussions, two concepts are introduced: normal and abnormal conditions. A “normal” condition means an average, stable or gradually changed condition where there is no sudden change in velocity, speed, energy or behaviours. In contrast, an “abnormal” condition means the condition where there is a sudden change.

Qualitative operators in QABGs correspond to the standard operators of real numbers, including addition, subtraction, multiplication, division, and equality: {+, -, \(*\), \(+\), =}. Qualitative values and qualitative operators are applied to QABG qualitative operations. Table 9 shows qualitative operations in QABGs. Assume \([--] \equiv [++]\) and \([-] \equiv [+].

\[ J = \{TF, TD, 0, 1, EC\} \]
Qualitative equations of QABGs

QABG qualitative equations provide the conceptual function to indicate the elements’ locations in the system structures and their individual behaviours as well as their interactions with the behaviour of the whole system. These equations contain constitutive relationships and all the necessary information about a physical system. For QABGs, numerical parameters in equations are replaced by symbols, i.e. qualitative values, which correspond to human states of incomplete knowledge.

Constitutive equations of QABGs in the following represent system structures and dynamic behaviours of people and variations of people-energy, or dynamic movements of building energy and variations of building energy:

- I, C, R elements associated with effort (e) and flow (f) variables, their relationships are, Eq (15)

\[
e = R \times f, \quad f = C \frac{d}{dt} e, \quad e = I \frac{d}{dt} f
\]  

(15)

- for elements of I, C, R, and CR, I-element stores p-variable, Eq (16), C-element stores q-variable, Eq (17), R-element consumes energy, Eq (18), and CR-element controls flow of energy, Eq (19). t is a sampling time period, where \( t_2 > t_1 \)

\[
\text{Inductor, } I: \quad e(t_2) = I \times (f(t_2) - f(t_1))
\]  

(16)

\[
\text{Capacitor, } C: \quad f(t_2) = C \times (e(t_2) - e(t_1))
\]  

(17)

\[
\text{Resistor, } R: \quad e(t) = R \times f(t)
\]  

(18)

\[
\text{Controller, } CR: \quad e(t) = CR \times f(t)
\]  

(19)

- for elements of TF and TD, the former receives either effort or flow information in one bond and generates the same information in another bond, Eq (20), and the latter establishes relationships between flow to effort and effort to flow, Eq (21)

\[
\text{Transformer, } TF: \quad e_{in}(t) = e_{out}(t), \quad f_{in}(t) = f_{out}(t)
\]  

(20)

\[
\text{Transducer, } TD: \quad e_{in}(t) = f_{out}(t), \quad f_{in}(t) = e_{out}(t)
\]  

(21)

- for elements of 0-junction and 1-junction

0-junction: the efforts attached on the bonds connected to a 0-junction are all equal and the algebraic sum of the flows is zero, \( \sum f = 0 \);

1-junction: the flows attached on the bonds connected to a 1-junction are all equal and the algebraic
sum of the efforts is zero, \( \sum e = 0 \).

In qualitative equations for a building subsystem, at any 0-junction or 1-junction, there are two qualitative equations representing relationships of effort (e) and flow (f) respectively at the 0-junction or the 1-junction and also, if any, the qualitative equations for different terminals connected at this 0-junction or the 1-junction.

### 4.3 QABGs model construction

A QABG model for a building combines graphical representations and qualitative equations. QABGs model construction for a building system starts from a building subsystem, to multi-subsystems of a building and then an integrated unified building system. An example is presented.

**Generating graphical representations**

A QABG graphical representation in a general form is generated by terminals (T) and junctions (J). Then, by replacing terminals (T) and junctions (J) with specific ABG elements, a precise QABG graphical representation is generated. After that, assign qualitative values to power variables of efforts and flows to each bond in the graphical representation.

- The generation of a single graphical representation for a building subsystem commences with identifying the source (S) of the building subsystem and dividing the building subsystem into one or more segments. It is followed by identifying every junction type, either 0-junction or 1-junction, between the segments and linking terminal element(s) of energy operators (T₀) to each junction. Then, depending on the building subsystem, TF or TD elements are linked to a 0-junction or a 1-junction, when the building subsystem involves energy transformation or transduction.

- The generation of multiple graphical representations for a number of building subsystems with energy interaction between them commences with generating a single graphical representation for each building subsystem. Energy controllers (EC) are then placed between the two graphical representations. Further, Link every energy controller (EC) to the 0-junction or 1-junction in the graphical representations of both building subsystems respectively where an energy interaction occurs.

- The unified graphical representation is generated by integrating multiple graphical representations
for numerous building subsystems with energy interaction(s) between them associated with energy controller(s).

Generating qualitative equations

Based on the graphical representations generated for different building subsystems of the building, the QABG qualitative equations for each building subsystem are generated.

- The generation of qualitative equations for a single graphical representation of a building subsystem commences from generating qualitative equations of power relations for the junction linked to the source and for the terminal elements of energy operators connected to this junction. Depending on the building subsystem, locate other junctions and generate qualitative equations for them and the connected terminal elements of energy operators. In additions, generate qualitative equations of power relations of TF or TD when energy transformation or transduction occurs.
- Generate qualitative equations for each single graphical representation of each building subsystem to form qualitative equations for multi-subsystems.
- Generate qualitative equations of each energy controller (EC) and link these qualitative equations with the qualitative equations of the building subsystems, the qualitative equations of the integrated unified building system are generated.

QABGs model construction for a building system

A small building with the space-people system and the lighting system in Fig. 7 is used as an example to show QABGs model construction of graphical representations and qualitative equations for a building system. In the space-people system, Fig. 7(a), there are two space-capacitors (C), rooms C1 and C2, and two controllers (CR), doors CR1 and CR2. In the lighting system, Fig. 7(b), there are a power source (S), controllers (CR), including switchboard CR1, and switches CR2 and CR3, as well as energy resistors, i.e. lights Ra and Rb.

The generation of a graphical representation for the space-people system commences from identifying the source (S) and divide the system into segments. A, B, and C are division points and also joints, Fig. 8(a). The next step is to identify the junction type as a 1-junction for joint A near the source (S). It is followed by linking the controller, door CR1, to the 1-junction identified, Fig. 8(b). Then, the processes
continues by finding the next junction and the connecting energy operators until the graphical representation for the space-person system is generated, Fig. 8(c). Finally, number each bond. Following similar processes, the graphical representation of the lighting system is generated, Fig. 9.

Energy interactions of the space-person system and the lighting system occur between room C1 and light a (Ra) as well as between room C2 and light b (Rb). Therefore, place energy controllers (EC) in between them. Further, link energy controllers (EC) to both building subsystems and integrate the graphical representations of two building subsystems into a unified graphical representation, Fig. 10.

The generation of qualitative equations for this space-person system is based on the QABG constitutive equations presented in Section 4.2. It commences with generating qualitative equations of power relations, i.e. effort (e) and flow (f), of the 1-junction linked by bonds 1, 2, and 3, Eqs. (22) and (23). A CR-element associated with bond 2 is linked to this 1-junction. This is followed by generating the qualitative equation for this CR-element, Eq (24), and then finding the other junctions within the space-person system, both 0-junctions and 1-junctions, sequentially. It is followed by generating qualitative equations of power relations for each junction and for the terminal elements of energy operators which are connected to this junction, Eqs. (25) to (31).

\[
e_1(nT) = e_2(nT) + e_3(nT) \tag{22}
\]

\[
f_1(nT) = f_2(nT) = f_3(nT) \tag{23}
\]

\[
e_2(nT) = CR_1 \times f_2(nT) \tag{24}
\]

\[
e_3(nT) = e_4(nT) = e_5(nT) \tag{25}
\]

\[
f_3(nT) = f_4(nT) + f_5(nT) \tag{26}
\]

\[
f_4(nT) = C_1 \times (e_4(nT) - e_4((n-1)T)) \tag{27}
\]

\[
e_5(nT) = e_6(nT) + e_7(nT) \tag{28}
\]

\[
f_5(nT) = f_6(nT) = f_7(nT) \tag{29}
\]

\[
e_6(nT) = CR_2 \times f_6(nT) \tag{30}
\]

\[
f_7(nT) = C_2 \times (e_7(nT) - e_7((n-1)T)) \tag{31}
\]

Following similar processes, the qualitative equations of the lighting system are generated, Eqs. (32) to (44).

\[
e_1(nT) = e_2(nT) + e_3(nT) \tag{32}
\]
Effort \( (e) \) and energy \( (E) \) variations in one building subsystem may cause variation of effort and energy in another building subsystem. Eqs. (45) and (46) are the qualitative equations generated for energy interactions between the space-people system and the lighting system. Eq (45) with energy controller EC1 is for the energy interaction between room C1 and light Ra. Effort \( (e) \) and energy \( (E) \) variations in the space-people system at door CR1, \( e_{2_{sp-rr}}(nT) \), and in room C1, including people within the room and people moving from room C1 to room C2, \( e_{4_{sp-rr}}(nT) \) and \( e_{5_{sp-rr}}(nT) \), may have energy interaction with light Ra, \( e_{6_{sp-rr}}(nT) \). This leads to effort and energy variations in the light system. In addition, Eq (46) with energy controller EC2 is for the energy interaction between room C2 and light Rb. Effort \( (e) \) and energy \( (E) \) variations in the space-people system at door CR2, \( e_{6_{sp-rr}}(nT) \), and in room C2, \( e_{7_{sp-rr}}(nT) \), may have energy interactions with light Rb, \( e_{9_{sp-rr}}(nT) \). This also leads to effort and energy variations in the lighting system.

\[
f_1(nT) = f_2(nT) = f_3(nT) \tag{33}
\]
\[
e_2(nT) = CR_1 \times f_2(nT) \tag{34}
\]
\[
e_3(nT) = e_4(nT) = e_7(nT) \tag{35}
\]
\[
f_3(nT) = f_4(nT) + f_7(nT) \tag{36}
\]
\[
e_4(nT) = e_5(nT) + e_6(nT) \tag{37}
\]
\[
f_4(nT) = f_5(nT) = f_6(nT) \tag{38}
\]
\[
e_5(nT) = CR_1 \times f_5(nT) \tag{39}
\]
\[
e_6(nT) = Ra \times f_6(nT) \tag{40}
\]
\[
e_7(nT) = e_8(nT) + e_9(nT) \tag{41}
\]
\[
f_7(nT) = f_8(nT) = f_9(nT) \tag{42}
\]
\[
e_8(nT) = CR_2 \times f_8(nT) \tag{43}
\]
\[
e_9(nT) = Rb \times f_9(nT) \tag{44}
\]

5. Qualitative simulation and inference using QABGs
QABGs can be applied to simulation and inference of people behaviours and people-energy variations in the space-people system, building energy movements and variations in the building energy system, and energy interactions between systems. People moving within the space-people system, and some building energy moving within the building energy system, can move in two directions: the same direction as and the opposite direction to people-flow or building energy flow moving from the outside into the building then into different rooms or devices. An ABG bicausal bond can represent people-flow moving in two directions and the resulting people-energy variations or building energy flow moving in two directions and the resulting building energy variations simultaneously.

Before applying QABG simulation and inference to different building subsystems and the building system, QABGs for people behaviours and people-energy variations in two-directional people movements are illustrated. This is followed by the depiction of QABGs for different types of energy interactions between the space-people system and the building energy system.

### 5.1 QABGs for people behaviours and people-energy variations in two-directional people movements

Applying QABGs for representing and simulating building energy movements and variations in two-directional building energy movements is very similar to applying QABGs for representing and simulating people-flow and unit people-energy variations in two-directional people movements. The following two cases of space-people systems, cases A and B, show the comparison of QABGs for people behaviours and people-energy variations in one-directional and two-directional people movements. The differences of graphical representations and qualitative equations between cases A and B are presented. Case B is for QABGs applied to people behaviours and people-energy variations in two-directional people movements.

**Case A**

People-flow moves from the corridor (I) through door CR2 into room B (C1) and moves through another door, door CR3, out of this room back to the corridor. Fig. 11 shows the plan drawing with people movement, Fig. 11(a), and the QABG graphical representation, Fig. 11(b). Eqs. (47) and (48) are the qualitative equations of the relationships of unit people-energy (e) and people-flow (f) at the 1-junction.
linked by bonds 1, 2, 3, 4, 5, and 11. People-flow moves in one direction from the outside into and remains in this building.

The movement of people-flow in this space-people system is at a normal speed. The qualitative values of each people-flow (f) attached to bonds 1, 2, 3, 4, 5, and 11 respectively and linked to the 1-junction are all equal, Eq (48). The variations of unit people-energy (e) caused by people-flow moving from the outside into this building, $e1(nT)$, and moving out of room B to the corridor, $e11(nT)$, are the inputs of the 1-junction linked by bonds 1, 2, 3, 4, 5, and 11. At this 1-junction linked by bonds 1, 2, 3, 4, 5, and 11, the qualitative value of the sum of unit people-energy inputs, $e1(nT)$ and $e11(nT)$, is equal to the qualitative value of the sum of unit people-energy outputs, $e2(nT)$, $e3(nT)$, $e4(nT)$, and $e5(nT)$, Eq (47).

$$e1(nT) + e11(nT) = e2(nT) + e3(nT) + e4(nT) + e5(nT)$$

(47)

$$f1(nT) = f11(nT) = f2(nT) = f3(nT) = f4(nT) = f5(nT)$$

(48)

**Case B**

People-flow moves from the corridor (I) through door CR2 into room B (C1) and moves through the same door, door CR2, out of this room back to the corridor. Fig. 12 shows the plan drawing with people movement, Fig. 12(a), and QABG graphical representation, Fig. 12(b). Eqs. (49) and (50) are the qualitative equations of the relationships of unit people-energy (e) and people-flow (f) at 1-junction linked by bonds 1, 2, 3, 4, and 5. ABG bicausal bonds 5, 7, and 8 represent people behaviours and people-energy variations in two-directional people movements, the same as and opposite to the direction of people-flow moving from the outside into the building.

Similar to case A, the movement of people-flow within this space-people system is normal. People-flow (f) moves from the outside into this building. The qualitative values of people-flow moving through door CR1 into the corridor (I) of the building and moving from the corridor through door CR2 into room B (C1) are all equal. However, people-flow moving through door CR2 out of room B to the corridor is moving in the opposite direction to people-flow moving from the outside into the building. For this people-flow, $f5(nT)$, the qualitative operator is subtraction, $-$, and the qualitative value is $[-]$. Eq (50). The variations of unit people-energy (e) caused by people moving from the outside into this building, $e1(nT)$ is the input of the 1-junction linked by bonds 1, 2, 3, 4, and 5. The qualitative value of the input,
$e_1(nT)$, is equal to the qualitative value of the sum of unit people-energy outputs, $e_2(nT)$, $e_3(nT)$, $e_4(nT)$, and $e_5(nT)$, associated with the unit people-energy variation caused by people-flow moving through door CR2 out of room B back to the corridor, $e_5'(nT)$. The qualitative operator of $e_5'(nT)$ is subtraction, $-$, and its qualitative value is $[-]$, Eq (49). The qualitative operation of $- e_5'(nT)$ implies that unit people-energy ($e$) in room B is decreased and unit people-energy in the corridor is increased.

$$e(nT) = e_2(nT) + e_3(nT) + e_4(nT) + (e_5(nT) - e_5'(nT))$$

(49)

$$f(nT) = f_2(nT) = f_3(nT) = f_4(nT) = f_5(nT) = - f_5'(nT)$$

(50)

In QABG qualitative equations, $eX'(nT)$ and $fX'(nT), X \in N$, are used to represent unit people-energy and effort of building energy ($e$), as well as people-flow and building energy flow ($f$) moving in the opposite direction to those moving from the outside into the building then into different rooms or devices.

The qualitative operator is addition, $+$, for people-flow and building energy flow ($f$), as well as the resulting unit people-energy variation and variation of effort of building energy ($e$) moving in the same direction as those moving from the outside into the building. The qualitative operator is subtraction, $-$, for people-flow and building energy flow ($f$) and the resulting unit people-energy variation or variation of effort of building energy ($e$) moving in the opposite direction to those moving from the outside into the building. Details of QABG qualitative values for representing and simulating people behaviours and people-energy variations in the space-people system as well as their interactions with building energy systems are presented in the following section.

5.2 QABGs for energy interactions between the space-people system and the building energy system

In the energy interaction between the space-people system and the building energy system, people-flow moving within a space-people system causes unit people-energy variation and variations in the need for different kinds of building energy. Building energy flow moving within the building energy system supplies building energy that people demand. Between the space-people system and the building energy system are the energy controllers (EC). People-flow moving within a space-people system and building energy flow moving within a building energy system can move in both the same direction and the opposite direction to people-flow or building energy flow moving from the outside into a building then
into different rooms or devices. However, the majority of building energy that people need and consume, such as electrical energy and hydraulic energy, is unidirectional and moves in the same direction as building energy moving from the outside into a building.

Fig. 13 shows different types of energy interactions between the space-people system and the building energy system. At the top of the figure are the six conditions of people behaviours and unit people-energy variations in the space-people system. At the bottom are the three conditions of building energy flow and variations of effort in building energy of the building energy system. Energy controllers are placed in between them. An energy controller (EC) receives the request for building energy needed from the space-people system. It triggers and also controls building energy activation and supply from the building energy system. QABG qualitative values apply to power variables, i.e. effort and flow, [-], [0], and [++] represent different abnormal behaviours, while [-] and [+] denote the normal behaviours.

In the space-people system, in conditions A, B and C, people-flow moves in the same direction as people-flow moving from the outside into a building then into different rooms. The movements of people-flow are all normal, \( f(nT) = [+] \). People-flow causes unit people-energy variations which cause variations in the needs of building energy and results in a request to the building energy system via an energy controller (EC). Unit people-energy variations in condition A are abnormal, \( e(nT) = [++] \), representing that people move into a room which causes sudden unit people-energy variations in the room they move into. In condition B, unit people-energy variations are normal, \( e(nT) = [+] \), representing that people are in a room and unit people-energy variations remain constant. Condition C represents people-flow moving out of a room in the same direction as people-flow moving from the outside into a building then into different rooms. Therefore, unit people-energy variations in the room decrease and are abnormal in the room they move out of, \( e(nT) = [--] \).

In conditions D and E, people-flow stops, \( f(nT) = [0] \). In condition D, no person is in the room and no unit people-energy variation is caused, \( e(nT) = [0] \). In condition E, there are some people in the room with normal unit people-energy variations, \( e(nT) = [+] \), which cause variations in the needs for building energy.

In condition F, people-flow moves in the opposite direction to people-flow moving from the outside into a building then into different rooms. It causes an abnormal unit people-energy decrease in the room where people move out of. The qualitative value is [-] for people-flow and [--] for unit people-energy
In the building energy system, there are three conditions, conditions a, b, and c, with connections to each condition in the space-people system. In conditions a and b, building energy systems receive requests from the space-people system and supply building energy to the space-people system by building energy flow. Building energy flow moving in the building energy system is normal, \( f(nT) = + \). Condition a represents that building energy supply commences or increases suddenly, the effort is abnormal, \( e(nT) = ++ \). In contrast, condition b represents that building energy supply remains constant, the effort is normal, \( e(nT) = + \). In condition c, building energy systems do not receive or respond to requests from the space-people system. Therefore, no building energy is supplied to the space-people system, there is neither building energy flow nor effort caused, \( f(nT) = 0 \) and \( e(nT) = 0 \).

6. Example

The following examples show how QABGs are applied to building simulations. The examples are worked out manually with a commentary to assist in their comprehension.

6.1 QABGs for simulation and analysis of building dynamics

A simple floor plan with a corridor, three rooms, and four doors is used as an example to show QABG simulation and inference of building dynamics with people behaviour and people-energy variations in a space-people system. Fig. 14(a) is the plan drawing. Its QABG graphical representation is shown in Fig. 14(b). Qualitative equations of this space-people system are in Table 10, Eqs. (51) to (71). They are applied in Case E.

People move in one direction from the outside into the building, then into different rooms. In this example, there is no person moving out of any room in the opposite direction to people-flow moving from the outside into the building. The simulation and inference of people behaviours and people-energy variations commence with applying qualitative values of normal and abnormal behaviours to power variables of people, i.e. unit people-energy (e) and people-flow (f), as well as to elements of I, C, R, and CR in the qualitative equations. This is followed by the inference of qualitative equations to complete the qualitative simulation to obtain every dependent variable, \( [d] \), in the qualitative equations.

Five cases, Cases A to E, of people behaviours and unit people-energy variations for this
space–people system are modelled to represent five different situations of dynamic people behaviours for this spatial arrangement. In these five cases, people move from the outside into this spatial arrangement. They move via the corridor into different rooms. They can move into one room, Cases A and B, Figs 15(a) and 15(b); into two rooms where they have an access to each other, Case C, Fig. 15(c); into two rooms where they have no access to each other, Case D, Fig. 15(d); or into these three rooms, Case E, Fig. 14. Controllers (CR), doors CR1 to CR4, control movements of people-flow. Graphical representations represented by ABG elements for each case are in Figs 14(b) and 15(a) to 15(d). Cases A, B, and C can be viewed as SISO systems (single input and single output), and Cases D and E as SIMO systems (single input and multiple outputs).

Details of simulations and inferences of people behaviours and people-energy variations of Case E are as follows, and listed in Table 10, Eqs. (51) to (71), in which doors CR1 and CR4 are all open:

- Assign qualitative value of normal behaviour, [+], to all component parameters in qualitative equations of this space–people system before applying inferential qualitative values into related components.

  People move from the outside, the source (S), through door CR1, via the corridor (I) into different rooms (C). They move at a normal velocity. Therefore, the qualitative value of people-flow ($f$) within the space–people system is [+].

  Rooms B, C, and D contain people. They remain normal, in the rooms. The qualitative values of C1, C2, and C3 are all [+].

  Corridor (I) stores people-impulse (p), and space-resistor (R) consumes people-energy. These behaviours remain normal. Therefore, the qualitative values for both I-element and R-element are [+].

- In the first inference, since doors CR1 to CR4 are all open, the qualitative values of doors CR1 to CR4 are all [0] which leads the qualitative values of $e_2$, $e_7$, $e_{10}$, and $e_{14}$ to [0], Eqs. (53), (60), (64), and (70).

  The qualitative values of people-flow as well as of I and R are [+]. Thus, the qualitative values of $e_3$ and $e_4$ are obtained as [0] and [+] respectively, Eqs. (54) and (55).

  From the corridor, people can move through door CR2 to room B and through door CR3 to room C, where there is no access to each other, Eqs. (56) and (57). People moving into room C can stay in
this room or move through door CR4 to room D, Eqs. (65) and (66).

When people move into different rooms, the qualitative value of unit people-energy variation caused by people-flow moving into a room is equal to the sum of unit people-energy varying around a door and in a room, Eqs. (58), (62) and (68). Since unit people-energy variations within rooms B, C, and D at time $nT$ are uncertain and depend on the current situation, thus the qualitative values of $e_8$, $e_{12}$, and $e_{15}$ are all dependent values, [d], Eqs. (61), (67), and (71). In addition, the qualitative values of $e_{13}$, $e_{11}$, $e_9$, $e_6$, $e_5$, and $e_1$ are dependent values, [d], as well, Eqs. (68), (65), (62), (58), (56) and (51).

- The next step is to infer the qualitative values of unit people-energy variations of $e_8$, $e_{12}$, and $e_{15}$, the output of this space-people system.

  The qualitative value of $e_8$ at time $((n-1)T)$, Eq (61), can be either [0] or [+], so can be $e_{12}$, Eq (67), and $e_{15}$, Eq (71). That is, the variation of unit people-energy can be zero or stable. The qualitative value of [+] at time $((n-1)T)$ is applied to $e_8$, $e_{12}$, and $e_{15}$.

  The moment of people moving into room B, unit people-energy variation within the room is increased suddenly, compared with the previous normal status within the room. It is abnormal. Assume that people move into the room during the period between time $((n-1)T)$ and time $nT$. Thus, the qualitative value of $e_8$ during the period between time $((n-1)T)$ and time $nT$ is [++] . It is equal to the qualitative values of unit people-energy variations caused by people-flow moving into rooms C and D, $e_{8(nT)} = e_{12(nT)} = e_{15(nT)} = [++]$. Therefore, the qualitative values of $e_{13}$, $e_{11}$, $e_9$, $e_6$, $e_5$, and $e_1$ are obtained as [++] .

  Fig. 16 shows the qualitative simulation and inference outputs of people behaviours and people-energy variations for people moving in one direction from the outside into the building, then into different rooms, during the time period from time 0 to time $nT$ in rooms B, C, and D including people-flow ($f$), unit people-energy ($e$), people-change ($q$), and people-energy ($E$), Figs 16(a) to 16(d). People-flow moves at a normal velocity, Fig. 16(a). Line (i) in Figs 16(b) to 16(d) represents the qualitative output for rooms B and D. People move into these rooms without anyone moving out of the rooms. In contrast, when people move into room C, they can stay or move out of the room to room D. The qualitative simulation and inference output for room C can be Lines (i), (iii), or in between Lines (i) and (iii), since the number of people moving into room C is always greater than, Line (i), or in between Lines
(i) and (iii), or equal to the number of people moving out of the room, Line (iii).

When usages of rooms are changed, QABG graphical representation and qualitative equations are changed correspondingly. By assigning qualitative values to the qualitative equations of the space-people system, qualitative simulation outputs of people behaviours and people-energy variations can be inferred and obtained. QABGs applied to the building energy system for simulation are similar.

In the next example, QABGs are applied to produce a simulation of people behaviours and people-energy variations in two-directional people movements in the space-people system, as well as for energy interactions between the space-people system and building energy system.

6.2 QABGs for building simulation and analysis of a building system

The spatial arrangement, the lighting system, and the hydraulic system of a building with five rooms are used as an example to show the application of QABGs for simulating energy interactions between a space-people system and building energy systems. Fig. 17 shows drawings of the spatial plan, lighting system, and hydraulic system.

QABGs simulation for people behaviours and people-energy variations in the space-people system

Again the simulation will be developed manually with accompanying commentary. In the space-people system, people move from the outside through door CR1 into room A of the building. From room A,

- some people move through door CR2 into and stay within room B and then close door CR2;
- some people move through door CR3 into room C. Some of them stay in the room and some of them move through door CR4 out of the room;
- some people move through door CR5 into room D. Some of them stay in the room and some of them move through door CR6 into room E. For people moving into room E, some of them stay in the room and some of them move out of the room through the same door, CR6.

Fig. 18 shows different people behaviours within the spatial arrangement of this building, Fig. 18(a), and the ABG graphical representation, Fig. 18(b). Qualitative equations representing the relationships of people behaviours and unit people-energy variations of this space-people system are generated in Table 11, Eqs. (72) to (100).
The simulation and inference of people behaviours and people-energy variations commence with applying qualitative values of normal and abnormal behaviours to power variables of people, i.e. unit people-energy (e) and people-flow (f), as well as to elements of C and CR, in the qualitative equations. Then, the dependent variables, [d], among the qualitative equations are evaluated sequentially. Details of simulations and inferences are as follows:

- Assign qualitative values of normal behaviour, [+], to the component parameters in the qualitative equations of this space-people system.

People-flow moves from the outside, the source (S), through door CR1 into this building. It moves at a normal velocity. Therefore, the qualitative value of people-flow (f) within the system is [+], Eqs. (73), (77), (81), (84), (87), (90), (93), and (96). Rooms A to E contain people and remain normal. The qualitative values of C1, C2, C3, C4, and C5 are all [+].

- In the first inference, doors CR1 to CR6, except door CR2, are open. The qualitative values of these elements are all [0] which leads to the qualitative values of e2, e9, e13, e16, and e20 to [0], Eqs. (74), (82), (88), (91), (97) and (98). Door CR2 is closed. Bond 5 represents that room B is not directly connected to the system. The qualitative values of CR2 and e6 are all [++] , Eq. (78).

People-flow (f) movements cause unit people-energy (e) variations. The resulting unit people-energy variations by people-flow movements are as follows. People move from the outside into room A (C1) of the building, Eq (72). From room A, Eqs. (75) and (76), some of them move into and remain in room B (C2), Eq (79). Some of them move through door CR3 into room C (C3), Eqs. (80), (83), and (86), and/or through door CR5 to room D (C4), Eqs. (89), (92) and (94), where there is no access to these two rooms. People-flow in room C can remain in the room or move through door CR4 out of this room, Eq. (86). People-flow in room D can remain in the room or move through door CR6 out of room D into room E (C5), Eq. (95); further, people-flow in room E can remain in the room or move through the same door, door CR6, out of this room, Eqs. (99), (100) and (95).

Since unit people-energy variations (e) within rooms A to E, C1 to C5, at time $nT$ are uncertain and depend on the current situation, the qualitative values of e3($nT$), e7($nT$), e11($nT$), e18($nT$), e21($nT$) and e21’($nT$) are all dependent values, [d], Eqs. (75), (79), (85), (94), (99) and (100). The qualitative values of e19, e17, e15, e14, e12, e10, e8, e4, and e1 are also dependent values, [d], Eqs.
The second and the third inferences are to obtain the qualitative values of unit-people-energy variations of $e_3$, $e_7$, $e_{11}$, $e_{18}$, $e_{21}$ and $e_{21}'$, the outputs of this space-people system. Assume qualitative values of $e_3$, $e_{11}$, $e_{18}$, and $e_{21}$ at time $(n-1)T$ are $[+]$, $e_7$ is $[0]$, and $e_{21}'$ is $[-]$. In the time period from $(n-1)T$ to $nT$:

People-flow moves into rooms A (C1), C (C3), D (C4), and E (C5), and unit people-energy variations within these rooms are increased suddenly, compared with the previous normal status within the rooms. Thus, the qualitative values of $e_3$, $e_{11}$, $e_{18}$, and $e_{21}$ in the time period are $[++]$.

People stay in room B (C2), unit people-energy variations within the room are increased normally. Thus, the qualitative value of $e_7$ in the time period is $[+]$.

People in room E move out of the room in the opposite direction to people-flow moving from the outside into the building. The qualitative operator is subtraction, $-$, and unit people-energy variations within this room are decreased suddenly, compared with the previous normal status within the room. Thus, the qualitative value of $e_{21}'$ in the time period is $[--]$.

Based on the qualitative values of $e_3$, $e_7$, $e_{11}$, $e_{18}$, $e_{21}$ and $e_{21}'$ obtained, the qualitative values of $[++]$ for $e_{19}$, $e_{17}$, $e_{15}$, $e_{14}$, $e_{12}$, $e_{10}$, $e_8$, $e_4$, and $e_1$ are inferred and obtained.

The qualitative simulation outputs produced for people behaviours and people-energy variations in the space-people system are then used for simulating energy interactions between the space-people system and the building energy system. The outputs of the space-people system become the inputs of the building energy system.

**Interactions between the space-people system and the building energy system**

ABG graphical representations for each building subsystem shown in Fig. 19, from the top to the bottom, are ABG graphical representations of the lighting system, the space-people system, and the hydraulic system respectively. Energy controllers (EC) are placed in between the space-people system and the lighting system, as well as between the space-people system and the hydraulic system. Linked by energy controllers (EC) attached with bonds, these graphical representations are integrated into a unified ABG graphical representation.

Processes for simulating and inferring building energy flow ($f$) and variations of effort of building
energy (e) in the lighting system and the hydraulic system are very similar to the processes for simulating and inferring people-flow and unit people-energy variations in the space-people system. However, building energy of the lighting system and of the hydraulic system is activated and supplied to respond to the needs in the space-people system. They are unidirectional and move in the direction of people-flow moving from the outside into the building then into different rooms. Eqs. (101) to (105) are the qualitative equations of the interactions between the space-people system and the lighting system:

- room A and light g with energy controller EC5, Eq (101):

\[
e_{2}(nT) + e_{3}(nT) + e_{4}(nT) = e_{109}(nT) \iff e_{110}(nT) = e_{26}(nT)
\]  

(101)

- room B and lights a, b and e, f with energy controllers EC1 and EC4, Eq (102):

\[
e_{6}(nT) + e_{7}(nT) = \begin{cases} 
    e_{101}(nT) & \text{EC1} \\
    e_{102}(nT) & \text{EC1} \\
    e_{107}(nT) & \text{EC4}
\end{cases}  
\]

(102)

- room C and light c with energy controller EC2, Eq (103):

\[
e_{9}(nT) + e_{1}(nT) + e_{13}(nT) + e_{4}(nT) = e_{103}(nT) \iff e_{104}(nT) = e_{13}(nT)
\]  

(103)

- room D and light h with energy controller EC6, Eq (104):

\[
e_{16}(nT) + e_{18}(nT) + (e_{19}(nT) - e_{19'}(nT)) = e_{111}(nT) \iff e_{12}(nT) = e_{29}(nT)
\]  

(104)

- room E and light d with energy controller EC3, Eq (105):

\[
(e_{20}(nT) - e_{20'}(nT)) + (e_{21}(nT) - e_{21'}(nT)) = e_{105}(nT) \iff e_{106}(nT) = e_{16}(nT)
\]  

(105)

Eqs. (106) to (110) are the qualitative equations of the interactions between the space-people system and the hydraulic system.

- room A and container d with energy controller EC10, Eq (106):

\[
e_{2}(nT) + e_{3}(nT) + e_{4}(nT) = e_{119}(nT) \iff e_{120}(nT) = e_{28}(nT)
\]  

(106)

- room B and containers a and f with energy controllers EC7 and EC12, Eq (107):

\[
e_{6}(nT) + e_{7}(nT) = \begin{cases} 
    e_{113}(nT) & \text{EC7} \\
    e_{114}(nT) & \text{EC7} \\
    e_{123}(nT) & \text{EC12}
\end{cases}  
\]

(107)

- room C and container e with energy controller EC11, Eq (108):
• room D and container b with energy controller EC8, Eq (109):

$$ e_{9}(nT) + e_{11}(nT) + e_{13}(nT) + e_{14}(nT) = e_{12}(nT) \iff e_{122}(nT) = e_{32}(nT) \quad (108) $$

• room E and container c with energy controller EC9, Eq (110):

$$ e_{16}(nT) + e_{18}(nT) + (e_{19}(nT) - e_{19'}(nT)) = e_{115}(nT) \iff e_{116}(nT) = e_{20}(nT) \quad (109) $$

In interactions between the space-people system and the lighting system, people move into every room and switch on the lights in every room at time \( t_1 \). Lights in every room are on at time \( t_2 \). In the lighting system, lights are R-elements and consume electrical energy. Current (\( f \)) demanded in the system is normal, \( [+\)\). Outputs of people-flow and unit people-energy variations in each room would not affect current movement and electrical energy variations. Fig. 20 shows qualitative simulation outputs of current (\( f \)), voltage (\( e \)), charge (\( q \)), and energy (\( E \)) of lights in every room.

In interactions between the space-people system and the hydraulic system, people move into every room as well as switch on the power and set the demand for the hydraulic power needed in each room at time \( t_1 \). Hydraulic energy in every room matches the requirement set at time \( t_2 \). In the hydraulic system, containers, which are also called terminal units, are C-elements that store \( q \)-variables and are controlled by energy controllers (EC) to interact with the space-people system. Water flow (\( f \)) moves within the system at a normal speed, \( [+\)\).

People-flow (\( f \)) moving into and remaining within a room in a period of time causes the variation of people-change (\( q \)), \( f \times t = q \). After people move into room B (C2), door CR2 is closed. People stay within room B. There is no person moving out of room B. People-flow and unit people-energy variations are normal. Then, when the hydraulic power is set and controlled to be a constant, the demand and supply of hydraulic energy increases steadily. Figs 21(a)-1, 21(b)-1, and 21(c)-1 show the qualitative simulation outputs of the hydraulic system of room B, including effort (\( f \)), Fig. 21(a)-1, change (\( q \)), Fig. 21(b)-1, and energy (\( E \)), Fig. 21(c)-1.

For people moving into and out of a room, the number of people moving out of the room can be less than or equal to the number of people moving into and within the room. When the number of people within the room changes, peoples’ demand for building energy can also change, such as in rooms A (C1),
C (C3), D (C4), and E (C5). The qualitative simulation outputs in Figs 21(a)-2, 21(b)-2, and 21(c)-2 are for the increasing demand of hydraulic power including effort (f), Fig. 21(a)-2, change (q), Fig. 21(b)-2, and energy (E), Fig. 21(c)-2. The energy controller increases the hydraulic power supply at time t3. The supply of hydraulic power reaches the requirement set at time t4. \( \theta \) represents the degree of variation. In Fig. 21(c)-2, \( \theta_i < \theta_j \) shows that the hydraulic energy variation after time t4 is more than it is during the period in between t2 and t3. In contrast, the qualitative simulation outputs in Figs 21(a)-3, 21(b)-3, and 21(c)-3 are for the decreasing demand of hydraulic power including effort (f), Fig. 21(a)-3, change (q), Fig. 21(b)-3, and energy (E), Fig. 21(c)-3. The energy controller decreases the hydraulic power supply at time t3. The supply of hydraulic power matches the requirement set at time t4. In Fig. 21(c)-3, \( \theta_i > \theta_j \) shows that the hydraulic energy variation after time t4 is less than it is during the period in between t2 and t3.

7. Conclusions

The energy-based unified representation for buildings, Archi Bond Graphs (ABGs), combining graphical representations and mathematical equations, has been developed. ABGs provide a unified approach to modelling and simulating different building subsystems and for representing and simulating their static and dynamic behaviours within the system structures. They have the capacity to represent two-directional people movements and people-energy variations within a space-people system. ABGs can also be applied to the simulation and analysis of interactions between space-people systems and building energy systems. When components or relationships are changed in any building subsystem of the space-people system or the building energy system, corresponding ones in related building subsystems will also be changed.

Qualitative Archi Bond Graphs (QABGs) draw on qualitative physics and discrete symbols to represent and simulate dynamic behaviours of building subsystems that provide a more general model applicable to a wide range of different conditions in buildings. QABG elements, element-link relationships, qualitative values and operations, and equations are developed in QABGs for generating graphical representations and qualitative equations.

QABGs were demonstrated to be applicable to building simulations for people behaviours and people-energy variations in a space-people system and their energy interactions with lighting systems and hydraulic systems. The qualitative simulation outputs of the space-people system become the inputs for
the lighting system and the hydraulic system respectively. QABGs simulations and inferences of the interactions in the case study show that people behaviours and people-energy variations in the space-people system may affect building energy supplied in building energy systems, depending on how building energy is consumed and stored in the building energy system.

QABGs have been shown to provide designers with a unified energy-based representation for communication, understanding, and evaluation of the interactions between different building subsystems. However, when QABGs are applied to a building system with more complex building subsystem-structures or building dynamics under time varying situations, the generation of the graphical representations and qualitative equations as well as the simulations and inferences become complex and increase in scale. The application of clustering and invariant embedding techniques are being explored as means to control both the growth of complexity and the scale effect so that the computations scale up linearly. This is left for a future paper.

Notations: Symbols of ABGs and QABGs

- 0 0-junction
- 1 1-junction
- \( \theta \) the degree of variation
- B building-component
- \( B_e \) exterior building-component
- \( B_i \) interior building-component
- \( BS \) building subsystem
- C capacitor
- CR controller
- E energy
- e effort
- EC energy controller
- f flow
- I inductor
- J QABG junction
L   ABG element
M   meter
P   power
p   momentum
q   displacement
QL  QABG element
R   resistor
S   energy source
Se  source of effort
Sf  source of flow
T   QABG terminal
t   time
Ts  terminal of energy source
To  terminal of energy operator
TD  transducer
TF  transformer

References


