CSI 771 / Stat 751 Midterm
Sample

Name (print) _____ G number _____

Work on this exam is to be done on your own. Please observe the Honor Code and certify that you have done so by signing the pledge.

Signature _____

Please put your answers on these pages. Write very carefully and legibly. *Show all work.*

1. (35) Let $Y_1, \ldots, Y_n$ be a random sample from a distribution with CDF P that has two unknown parameters θ and λ. We can write θ as $\Theta(P)$ and λ as $\Lambda(P)$.

   For simplicity, let us assume θ and λ are scalars.

   Our interest is in using the sample to make inferences about θ. There are various ways of doing this.

   In each of the following questions, do not forget also to tell what to do about λ.

   (a) How would you estimate θ using the plug-in principle?

   (b) How would you estimate θ using maximum likelihood?

   (c) How would you estimate θ using least squares?

   (d) How would you form a 95% confidence set for θ? (Specify all of the things needed to do this.)

   (e) In the preceding question, you may not have everything that is needed. Describe one way you could use the bootstrap to form a 95% confidence set for θ.

   (f) Describe another way (different from your answer in the preceding question) to use the bootstrap to form a 95% confidence set for θ.

(Continued. Use all the definitions on the previous page.)

(g) How would you do a Monte Carlo test, at the 0.05 level, of the null hypothesis that $\theta = 100$ against the alternative that $\theta \neq 100$? (Don't forget about λ.)

(h) Suppose you have a statistic $T(Y)$ to use as an estimator of θ. You need the variance of $T(Y)$. Describe the steps to use the jackknife to estimate the variance of $T(Y)$.

(i) Describe the steps to use the bootstrap to estimate the variance of $T(Y)$.

(j) Suppose $\Theta(P) = \theta$ is such that

$$\int_{-\infty}^{\theta} \mathrm{d}\, P(t) = 0.5.$$

What is the plug-ing estimate of θ?

2. (15) Consider the problem of estimating θ in a uniform distribution over $[0, \theta]$. From a sample of size n, an estimator is $T = x_{(n)}$, the maximum order statistic.

(a) Show that T is biased.

(b) Derive (describe) the nonparametric bootstrap bias-corrected estimator for θ.

3. (15) Assume you have a source of uniform U(0, 1) random numbers and you want to generate random numbers from the distribution with density

$$\begin{aligned} p_X(x) &= \frac{3}{2}x^2, \quad \text{for } -1 \le x \le 1, \\ &= 0, \quad \text{otherwise.} \end{aligned}$$

(This is a type of beta distribution.)

(a) Write out the exact steps you would use to do this by the inverse CDF method.

(b) Write out the exact steps you would use to do this by the acceptance/rejection method. Clearly identify any function or constant that you use.

4. (15) Describe how you would evaluate these integrals using Monte Carlo. (Give the formulas, and tell what kinds of variables you use.)

(a) $\int_{-1}^{1} \cos(x)x^2 \mathrm{d}x$

(b) $\int_0^2 \int_0^\infty y \cos(x)e^{-x} \mathrm{d}x\mathrm{d}y$

5. (20) Given the vector $x = (2, 3, 4)$.

   (a) What is the matrix of the transformation that will rotate x into the vector $(y_1, y_2, 0)$?

   (b) What are y_1 and y_2? (Give their numeric values.)

   (c) Given the vector $z = (1, 1, 1)$. Determine a transformation of z that makes it orthogonal to the vector $(y_1, y_2, 0)$ above.

   (d) Is your transformation in Part (c) linear? orthogonal? isometric? affine? projective? (The answers may depend on the specific transformation you use.)

   (e) What is the Euclidean distance between x and z above?