Lab 10 – The Photoelectric Effect

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Abstract

We experimentally derive Planck's Constant, denoted $\hbar$, by measuring photocurrent while varying reverse voltage applied to a phototube illuminated by several Light Emitting Diodes. By simple graphical analysis of the collected data we found $\hbar$=6.60E-34 with an uncertainty of ± 4%. Applying a more advanced analysis, modeling the phototube current-voltage relationship as a vacuum diode, measurement uncertainty was significantly reduced. This method found $\hbar$=6.670E-34 ± 0.8%. Both methods produced values accurate to within 1% of the accepted value for the constant ($\hbar$=6.62607E-34). While this experiment may be considered a success, methods to greatly simplify future experiments are also presented.

Introduction

The nature of light has been contemplated since the earliest written history. References to light as particles are found in ancient Greek pre-Socratic and Indian Hindu teachings as early as the 6th - 5th century BC. The Rigveda, a collection of Indian hymns thought to be composed between 1700BC-1100BC[1], was the first to decompose the visible spectrum into primary colors. The Greek philosopher Euclid made the first formal studies of geometric reflection 300 BC, and Ptolemy wrote about the refraction of light in the 2nd century AD.

Scientific investigation in the modern field of optics began with René Descartes in 1637, who studied refraction and proposed that light was akin to sound waves. Isaac Newton published his particle theory of light in 1704 in rebuttal to Descartes, on the basis that light travels in straight lines geometrically[2] rather than bending around obstacles like plane waves in water or the compression waves responsible for sound in air[3].

Newton's theory stood unchallenged for a century, until Augustin-Jean Fresnel 1807 experiment on diffraction[4] seemed to prove the wave-like nature of light was accurate. Fresnel empirically developed a set of equations which accurately described the refraction of light, with geometric properties based on propagation mode of the wave. In 1861, James Clerk Maxwell essentially defined classical EM by unifying Gauss's law, Faraday's law and Ampere's law, into (what is known today as) the Maxwell Equations. Maxwell solved the electromagnetic wave equation for light[5], which would seem to only be possible if it were a wave.

In 1887, Heinrich Hertz published his observations on the production and reception of electromagnetic waves in the journal Annalen der Physik. In his article, he briefly mentioned a result where the potential required to make a spark jump a particular air gap was lower when the electrodes were illuminated with ultraviolet (UV) light. At the time however, Maxwell's wave theory of light had been

1 http://en.wikipedia.org/wiki/Rigveda#cite_note-3
2 As repeated in Experiment 6 of Physics Lab 263
3 http://en.wikipedia.org/wiki/Isaac_Newton#Optics
4 As repeated in Experiment 8 of Physics Lab 263
5 Maxwell, James Clerk (1865). "A dynamical theory of the electromagnetic field". Philosophical Transactions of the Royal Society of London 155: 459–512. doi:10.1098/rstl.1865.0008

generally accepted for 40 years, and Hertz discounted the result as anomalous.

At the time, many scientists were using evacuated tubes to study the nature of electrons emitted from a metal surface[6], and Hertz's result spurred a series of experiments on the effects of radiation on the electron emission. One such experiment was performed by Phillip Lenard in 1902. By measuring the current through a phototube - an evacuated glass tube containing a metal plate (cathode) and metal wire (anode) separated by a short distance - Lenard found photocurrent to be directly proportional to illuminating intensity; but the kinetic energy of the individual electrons (equal to the voltage required to stop all photocurrent) was proportional to the frequency of the light. In other words, when illuminated by UV radiation a higher voltage across the tube was required to stop photocurrent than when illuminated by blue or green light.

Lenard's results seemed to conflict with Maxwell's wave theory, which explained the photoelectric effect by electron resonance; If Maxwell was correct, like an antenna at radio frequencies, the energy of escaping electrons should scale with intensity, independent of frequency. In 1905 Albert Einstein, in an attempt to resolve the apparent paradox, postulated light was composed of discrete quanta (i.e. photons) rather than continuous waves. Based on the constant $\hbar$ introduced in 1900 by Max Planck in his law of blackbody radiation to resolve the ultraviolet catastrophe[7], Einstein theorized that energy of a photon is a multiple of its frequency such that $E=\hbar f$ - a theory which created the field of quantum physics and won him a Nobel Prize. This result led Einstein to develop the theory of wave-particle duality, finally ending the heated 7000 year debate between the wave and particle light theory proponents.

One result of Einstein's theory is that an absorbed photon must transfer exactly $\hbar f$ energy, which explains the photoelectric effect – regardless of the number of photons (intensity), the energy of electrons released during PE interaction is always $\hbar f$, dependent only on the frequency of light. In this experiment, we attempt to find the value of $\hbar$ using a procedure similar to that used by Lenard, by measuring voltage bias required to eliminate photocurrent of a phototube illuminated by several different wavelengths of light.

6 Most famously, in 1899 J. J. Thompson performed an experiment to determine the electron charge-mass ratio (as repeated in Lab 4), showing that an electron was much smaller than the atom.

7 This is an extremely interesting subject which was fundamental the credibility of quantum theory. In short, the issue was when defining the spectral emission of an object using a continuous variable, the object will emit infinite energy as its temperature increases – an entirely unrealistic physical impossibility. see en.wikipedia.org/wiki/Ultraviolet_catastrophe

Apparatus

For this experiment, we used a CENCO TY-7 “Self-Contained Planck’s Constant Apparatus”[8] consisting of a sealed enclosure with a 1P39 type phototube, 0-2V bias supply, and 0-100µA current amplifier with zero-point adjust. Two Meterman 38XR digital multimeters (M1, M2) were used for current and voltage measurements. A 10V DC power supply was used to power LEDs through a 330Ω resistor which illuminated the phototube at various wavelengths, including red (695nm specification), yellow (587 nm specification), green (565 nm specification) and blue (430 nm specification). A black cloth was used to ensure minimal stray light entered the apparatus during each run.

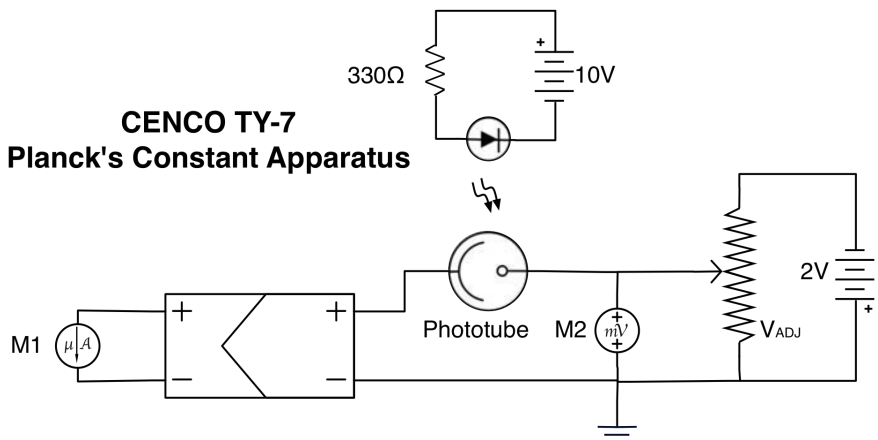


Figure 1: Apparatus Schematic

Procedure

Five trials were conducted. Four, one for each color, were conducted at 20mA LED current. One additional trial was conducted with the blue LED set to 5mA. A later trial was conducted with a much brighter red LED at 350mA drive. For each trial, the appropriate LED was configured in the apparatus LED socket. With the LED power supply off, the TY-7 amplifier current was adjusted to zero. After turning the LED supply on, blocking voltage was varied and photocurrent was measured using the 38XR meter at four points of decreasing reverse voltage. This was repeated five times for each LED, producing 20 data points per wavelength. Data was recorded in Excel.

8 http://www.cencophysics.com/self-contained-plancks-constant-apparatus/p/IG0041905/

Data

The one hundred data points collected during the five trials are plotted below, on linear (Figure 1) and logarithmic (Figure 2) voltage scales.

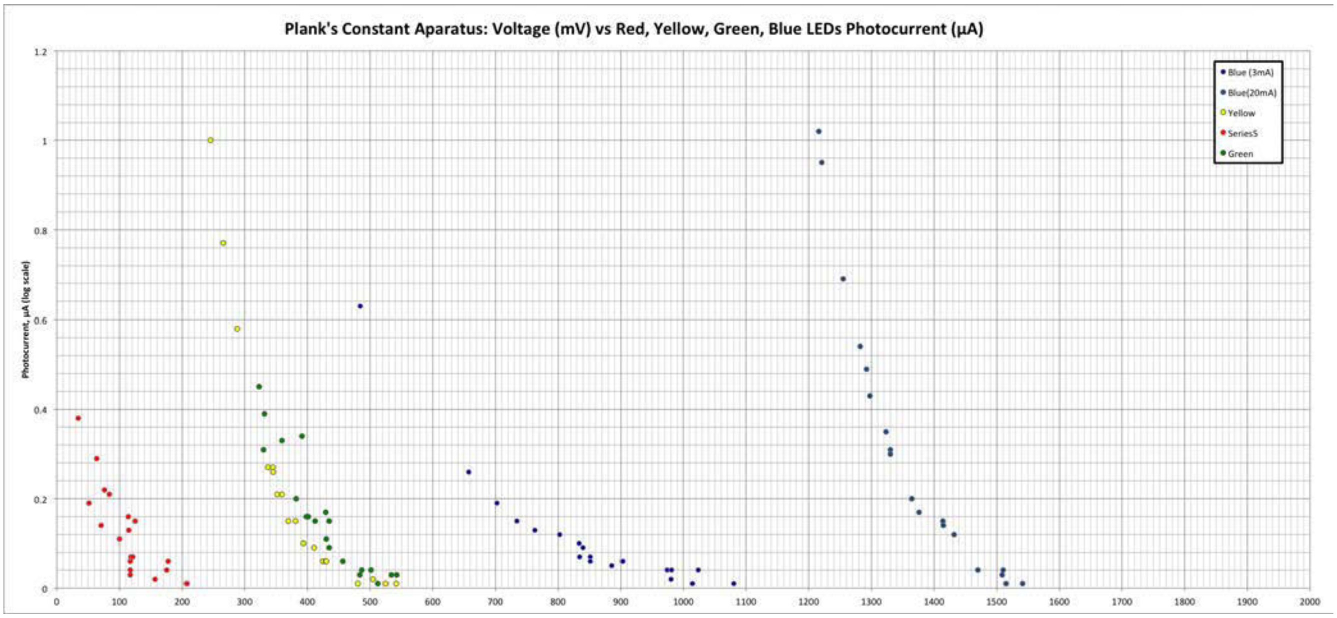


Figure 2: Linear Current Axis Plot of Experimental Data

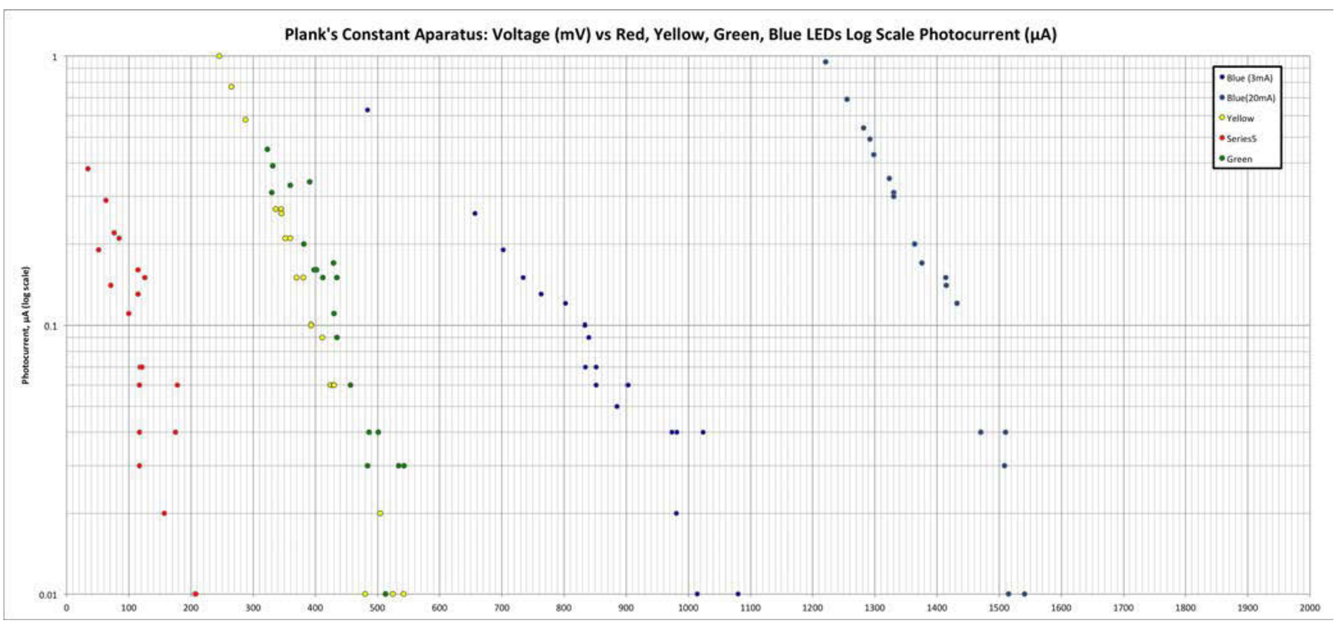


Figure 3: Logarithmic Current Axis Plot of Experimental Data

Analysis

Graphical Estimation of V_0

The most trivial approach to analysis is graphically estimating the V_0 intercept. Four methods to extrapolate the zero point were pursued, with results presented in Table 1. Methods are explained below.

λ (nm)	*f* (Hz)	V_0 (V) Method 1	V_0 (V) Method 2	V_0 (V) Method 3	V_0 (V) Method 4
697 nm	4.301E+14	0.3200 V	0.1650 V	0.3650 V	0.1526 V
587 nm	5.107E+14	0.5550 V	0.4100 V	0.5575 V	0.4435 V
565 nm	5.306E+14	0.6050 V	0.5300 V	0.6600 V	0.4902 V
430 nm	6.517E+14	1.1750 V	0.9500 V	1.2550 V	0.8470 V
Confidence		84%	69%	92%	67%
e/ħ Experimental		3.920E-15	3.575E-15	4.122E-15	3.103E-15
ħ Experimental		6.280E-34	5.727E-34	6.604E-34	4.971E-34
Experimental Error		-5.22%	-13.57%	-0.34%	-24.98%

Table 1: Graphical Estimation Summery

1. A full bounds logarithmic fit. The logarithm of photocurrent was plotted against voltage. Lines were drawn down the upper and lower bounds of the data points for each color. Uncertainty was taken to be the distance between upper and lower bounds, the value was taken to be the average of upper and lower bounds.

2. Full bounds linear fit. Linear photocurrent was plotted against voltage. Lines were drawn down the upper and lower bounds of the data points for each color. Uncertainty was taken to be the distance between upper and lower bounds, the value was taken to be the average of upper and lower bounds.

3. Logarithmic fit, current less than 100µA. Same as 1. except data below 100µA was ignored.

4. Linear Regression Fit. Data was fit using the LINEST function in Microsoft Excel. Uncertainty was taken to be the RMS R^2 error.

Child-Langmuir Vacuum Diode Fit Analysis

The phototube used in this experiment is an unusual form of a vacuum diode, the model for which was derived independently by Child in 1911 and Langmuir in 1913. It is known now as the Child-Langmuir equation or the three-halves power law:

$$I_b = k\,E_b^{3/2} \qquad (1)$$

where $k=\frac{2.33\times10^{-6}}{d^2}A$ for parallel plates with area A separated by distance d ; or

$14.68\times10^{-6}\frac{L}{r_a\beta^2}$ for coaxially arranged electrodes with length L and anode radius r_a , where β an efficiency factor proportional to the ratio of anode to cathode radius that tends to 1 for small cathodes surrounded by large anodes.

Happell [9] offers insight into empirically characterizing vacuum diodes. Using the apparatus depicted below:

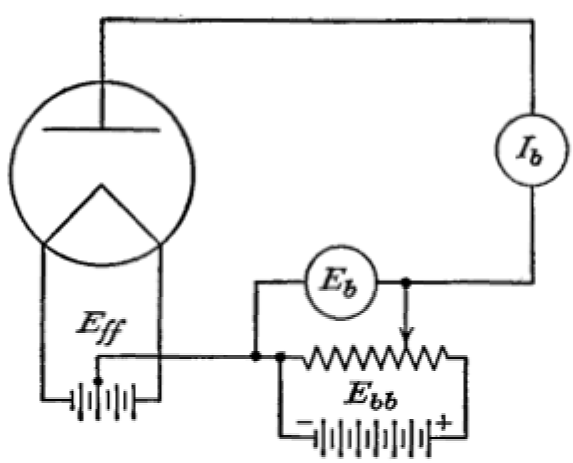


FIG. 3-7. Circuit used to evaluate the constants of the Child-Langmuir equation.

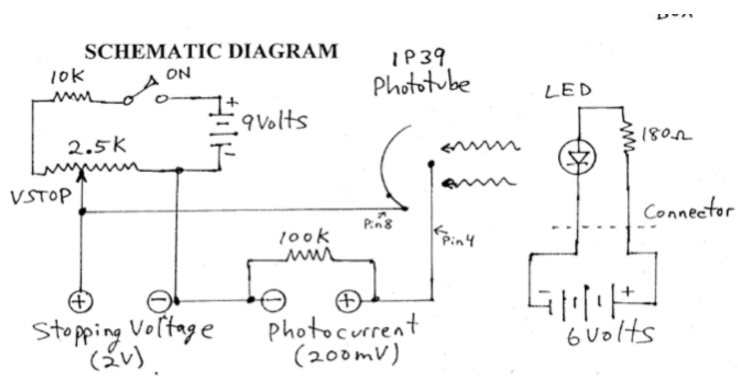
9 Happell, G., "Engineering Electronics", McGraw Hill; 1953 New York. 53-5166. ch 3

He suggests assuming that the diode follows a power law, but not necessarily with a three-halves exponent, in the form $I_b = k E_b^n$. This can be written in logarithmic form as:

$$\log I_b = \log k + n \log E_b \quad (2)$$

which is an equation for a straight line if $\log I_b$ is plotted vs $\log E_b$. When plotted, the y intercept (i.e. $\log E_b = 0$) is the value k , and the slope of the line is the constant n .

Note the similarities to of the figure above to the sort of apparatus used in our experiment[10],



With the only difference being that we apply a *negative* voltage to the anode, rather than a positive one. We are unlikely to be successful in fitting a given k because one of the assumptions required to derive (1) is zero initial velocity for electrons immediately above the surface of the cathode (which are then drawn towards the anode via a positive potential gradient (c → a)). In our phototube, the potential is due to the photoelectric effect and *all* electrons leaving the surface of the cathode have non-zero velocity ($\sqrt{2 m_e \hbar c/\lambda - W}$). However it is worth perusing (2), to determine a value for $\hbar$.

The energy imparted to a dislodged electron is at most $eV = \hbar f - W$, where W is the work function of the cathode material. When zero external potential is applied to our phototube, we postulate that the conditional equivalence to the space charge created in a thermionic vacuum with a potential gradient of V . To determine the stopping voltage V_0 , the V-I plot should fit:

10 Garver, Wayne. "LED Photoelectric Effect Apparatus", University of Missouri, St. Louis (PDF)

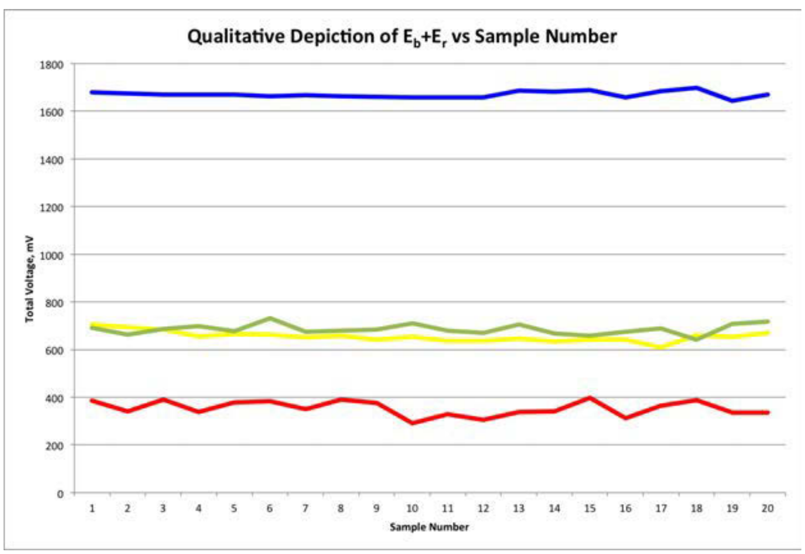
$$\log I_b = \log k + n \log(E_b + V_o) \quad (3)$$

Using a regression technique, the work function (combined with bi-metallic potential difference) of the tube was fit to be $W = 1.44\text{eV}$. Using this combined with (3), the best fit for the tube was found to be $N = 1.57$, slightly higher than the 1.5 predicted by Child-Langmuir but still remarkably close considering that most of the assumptions under which it was derived have been broken. A higher value makes sense, as our phototube is bound to be less efficient than a purpose built rectifying diode. Best fit for the constant was $\log k = -4.78$, i.e. $k = 0.00839$.

From (1) we find may find the bias voltage as a function of space-charge limited current to be:

$$E_b(I_b) = \exp\left(\frac{\log I_b - \log k}{n}\right) \quad (4)$$

from which we expect that $E_b(I_b) + E_R = \frac{(\hbar f - W)}{e}$, that is $E_b(I_b) + E_R = V_0$. To illustrate this qualitatively, the 100 data points are again plotted below, after adding $E_b(I_b) + E_R$:



More concretely, when propagated through the following values for $\hbar$ were determined were all under ±1% error, to 1.67% uncertainty. Values for each trial are individually listed in Table 2, and all trials are plotted in Figure 4.

Trial	V_b+V_r + W (mV)	ℏ, Experimental	Error
Red	1793.67 ± 31 mV	6.68139E-34	-0.835%
Yellow	2094.85 ± 21 mV	6.57176E-34	0.820%
Green	2125.39 ± 21 mV	6.56534E-34	0.917%
Blue	3109.84 ± 18 mV	6.64796E-34	-0.330%

Table 2: Vacuum Diode Equation Fit Summery

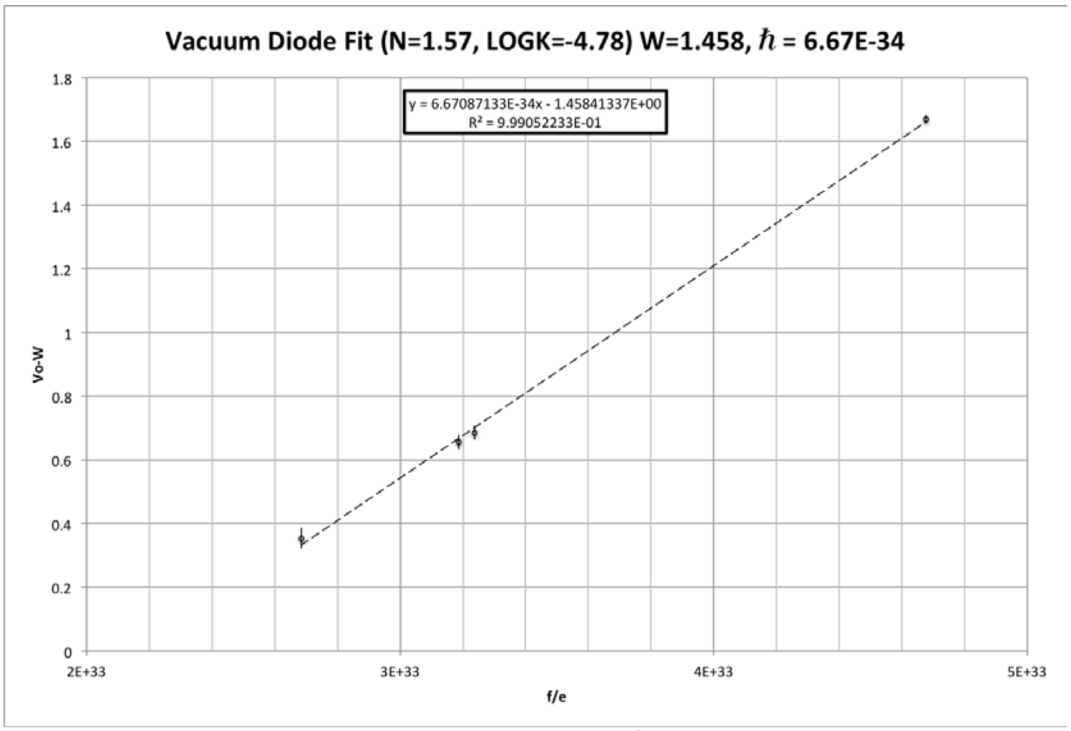


Figure 4: Plot depicting linear fit where slope is ℏ, intercept is W

Discussion

The procedure we employed obtains ℏ in a very roundabout way. The TY-7 apparatus was designed for use with a filtered mercury vapor lamp; choosing various transition lines in the mercury spectrum via a diffraction grating[11]. This is a reasonable approach because:

1) The exact spectral signature of elemental mercury can be exactly calculated (via Rydberg, the most precisely measured constant in physics) which we have demonstrated in a previous lab.[12]

11 http://www.physics.usyd.edu.au/~kev/intermediate_lab_manual/a3.pdf
12 See Lab 9

2) The transition lines are wide enough apart to be separated by commodity gratings, while still being visually identifiable. In other words, it is relatively straight forward to ensure the correct line is illuminating the phototube.

3) A 100W or 200W mercury vapor lamp produces substantial intensity in all transition bands, minimizing small-scale error and parasitic effects by ensuring the space-charge in the phototube is saturated for each trial

4) There are four (or 5, if the UV band can be used by watching the ammeter at zero bias) wavelengths from 578nm down to 404nm (or 365) which are enough for a decent least squares fit with low uncertainty in (1)

However, for reasons unbeknownst to the author, the designers of our apparatus substituted four light emitting diodes in place of the Hg lamp and grating. Presumably, the assumption was that the four spectral lines could be cheaply and cleanly replaced by the four monochromatic LEDs with negligible effects on the results. This is a faulty assumption however because:

1) There is no way to calculate the exact spectral signature of an LED with available information; nor is it consistent from one device to the next. LED wavelength depends not only on junction semiconductor choice, but also on doping, physical geometry, temperature, drive current, potting material, and drifts over the lifetime of the device at a rate proportional to drive current. Depending on material, the dominant wavelength can vary between 10nm (miniature) and 40nm (high power) between two devices[13]. It would be difficult to measure the spectral signature of an LED because it is inherently monochromatic, there are no neighboring transition lines to compare, requiring an absolute spectrometer.

2) For the reasons outlined in (1), it is impractical to achieve better than 50nm estimate on wavelength when using LEDs (regardless of “what's on the box”). The aging factor further complicates the situation. The main advantage to the Hg lamp is the spectral signature does not

13 In fact, this is a serious problem in the LED display industry. High end manufactures “bin” LEDs, measuring wavelength post-assembly and selling like-wavelength units under a specific part number. The LEDs used in this apparatus were not binned.

change over time, drive current, etc... A blatant demonstration of this is the data shown in Figure 2, where the same blue LED was driven at 3mA and 20mA, with a V_0 difference of 50%.

3) Inexpensive LEDs, like those used in our apparatus, provide only a fraction of the intensity of an Hg lamp. As such, small-scale measurement errors become a major issue. The majority of data taken during this experiment was in the 10μA range, below the 0.1mA precision of the DMM used to collect it. With a source 100 times brighter, the "zero" point could safely be taken as anything below 1mA of photocurrent, which would provide the same (or better) precision while eliminating the need for complex correlation analysis. Brighter LEDs are available, but are more susceptible to the issues outlined in (1) and (2) at high drive currents. They are also much more expensive, potentially negating the benefit of replacing the Hg lamp.

4) With the significant uncertainty introduced by the three points above, four data points is no longer sufficient for a least-squares fit. Others (see [10]) have suggested using up to 8 LEDs, which is one way to address the error. Taking more data, and using the method proposed in [9] is another possibility.

In summery, using a series of LEDs to illuminate a vacuum phototube is probably the worst possible way to perform this experiment. Interestingly enough, however, the availability of LEDs presents an opportunity in itself for an alternate means of determining $\hbar$.

In a semiconductor junction, a specific potential is required to push electrons from their initial state in the valance band to an excited state conduction band before current will flow. The amount of energy required is known as the band gap, measured in eV. An excited electron will eventually fall back into the valance band ("into a hole"), and by conservation of energy, release the same amount of energy it received jumping the gap.

Rectifying diodes, transistors, and integrated circuits are made of so called "indirect" band gap materials, where the two states have different k-vectors, meaning the electron is much more likely to fall into an intermediate state than directly back to the valence band. In such materials, the majority of recombination energy is transferred to the crystal lattice by increasing it's inertia (raising the temperature).

Optical semiconductors, like LEDs, use semiconductors blends chosen for a high, direct band gap. In direct band gap materials, the excited and rest states have the same k-vector, meaning that recombination is likely to produce a photon rather than heat the lattice. The potential energy released by the recombination event is conserved by producing a photon with the same kinetic energy. By wave-particle duality, the emitted photon frequency is proportional to the band gap energy by $eV = \hbar f$. This relationship is illustrated in Figure 5 for several semiconductor materials.

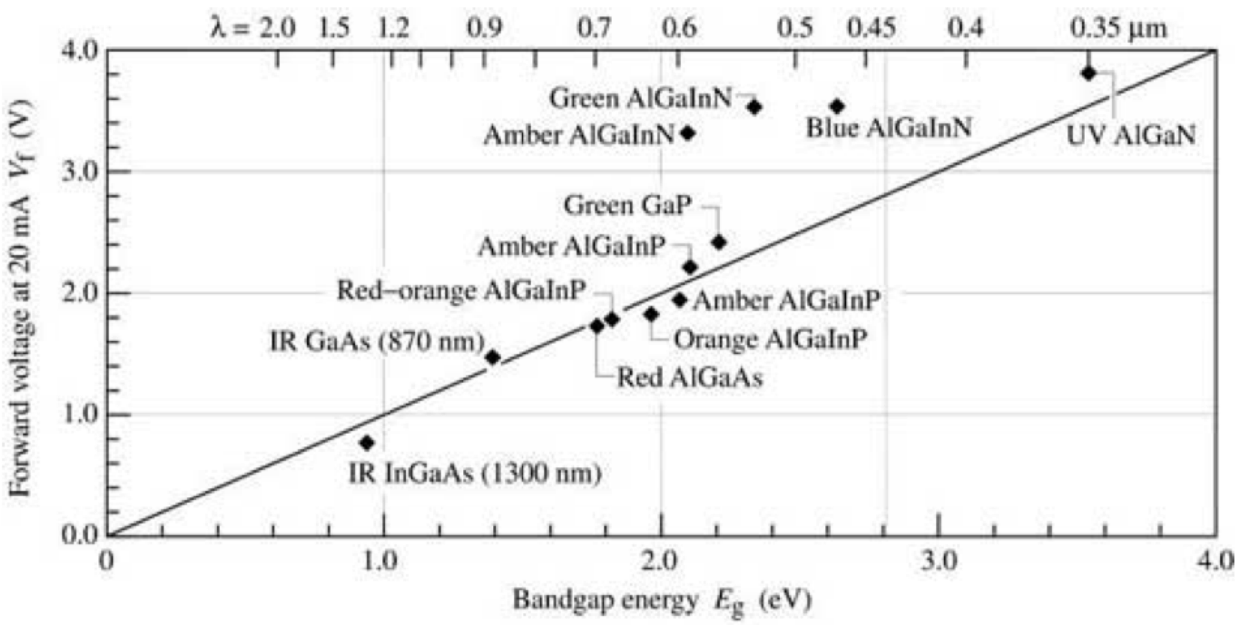


Figure 5: LED Forward Voltage vs Bandgap, λ (http://lcd.creol.ucf.edu/OSE6820/LED.pdf)

Thus, in theory, if one knew (as assumed in this lab) the wavelength of several direct band gap LEDs, or could make a reasonably accurate measurement of their emission, it is possible to determine $\hbar$ with a single variable voltage supply, a volt meter with mV accuracy, and an ammeter with µA accuracy. **That is, keeping the rest of the apparatus described above, the entire TY-7 phototube unit is unnecessary for this experiment and only adds error and confusion to the measurement process.** Indeed, it appears that CENCO has ceased production of the TY-7 in favor of the much simpler, less expensive, easier to maintain LED-only version[14] which features seven LEDs to address concerns in

14 http://www.cencophysics.com/plancks-constant-determination-box/p/IG0041908/

point (4) above.

In reality, there are some parasitics which prevent perfect correlation - the nature of which are laboriously reviewed in [15],[16], however highly accurate, relatively simple models for LEDs are widely available and current, unlike the now-defunct vacuum tube diode model referenced in [9]. The overall uncertainty in an LED-only experiment is much lower, due to factors listed in Table 3.

Just LEDs	Phototube + LEDs
• Must be direct-band gap • Equivalent series resistance • Equivalent parallel/bypass resistance • LED temperature	• Current amplifier zero setting (q current) • Current amplifier linearity • Illumination intensity • Residual plate charge (time dependence) • Cathode coating work function • Cathode coating spectral response • Cathode-anode junction potential • Phototube temperature

Uncertainty in Both Methods
• Uncertainty in LED wavelength • LED Wavelength dependence on drive current • Measurement uncertainty (DMM)

Table 3: Uncertainty in LED vs Phototube+LED experiments

Conclusion

In this experiment, we have derived Planck's Constant $\hbar$ by measuring photocurrent through a phototube, while varying reverse voltage. When the tube was illuminated by several wavelengths of reasonably monochromatic light provided by an assortment of LEDs (Light Emitting Diodes), each wavelength required a different reverse voltage to prevent current flow. By simple graphical analysis of the collected data we found $\hbar$=6.60E-34 with an uncertainty of ± 4%. Applying a more advanced analysis, modeling the phototube current-voltage relationship as a vacuum diode, measurement uncertainty was significantly reduced. This method found $\hbar$=6.670E-34 ± 0.8%. Both methods produced values accurate to within 1% of the accepted value for the constant ($\hbar$=6.62607E-34). JSD

15 http://lcd.creol.ucf.edu/OSE6820/LED.pdf
16 http://www.ecse.rpi.edu/~schubert/Light-Emitting-Diodes-dot-org/