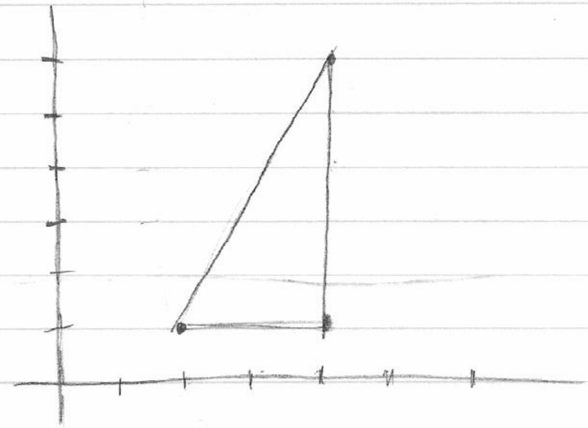
John Beauregard

CS 485 Homework # 1

1.



a) $\det(T_1) = \cos^2\theta + \sin^2\theta$

b) $\det(T_2) = -\sin^2\theta - \cos^2\theta$

yes, orthonormal

c) yes, it results in a clockwise rotation as opposed to a counterclockwise rotation

2. $P_A = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$

rotation over Z axis by θ

$$P_A' = \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

rotation over x axis by ϕ

$$P_A'' = \begin{bmatrix} P_1'' \\ P_2'' \\ P_3'' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$P_A'' = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \cos\phi\sin\theta & \cos\phi\cos\theta & -\sin\theta \\ \sin\phi\sin\theta & \sin\phi\cos\theta & \cos\theta \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

3. $\theta = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right)$

$$\omega = \frac{1}{2\sin(\theta)} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

Given $R = \begin{bmatrix} .1729 & -.1468 & .9739 \\ .9739 & .1729 & -.1468 \\ -.1468 & .9739 & .1729 \end{bmatrix}$

$$\theta = \cos^{-1}\left(\frac{3(.1729) - 1}{2}\right) = \cos^{-1}(-.24065) = 103.925$$

$$\omega = \frac{1}{2\sin(103.925)} \begin{bmatrix} .9739 + .1468 \\ .9739 + .1468 \\ .9739 + .1468 \end{bmatrix}$$

$$\omega = \frac{1}{1.94122} \begin{bmatrix} 1.1207 \\ 1.1207 \\ 1.1207 \end{bmatrix} = \begin{bmatrix} .5773 \\ .5773 \\ .5773 \end{bmatrix}$$

$$\omega = \begin{bmatrix} .5773 \\ .5773 \\ .5773 \end{bmatrix} \qquad \theta = 103.925^\circ$$

4.

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \\ 1 \end{bmatrix} = T_{AB} \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} = \begin{bmatrix} R_{AB} & t_{AB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} \qquad t_{AB} = \begin{bmatrix} X_B \\ 0 \\ 0 \end{bmatrix}$$

$$R_{AB} = \begin{bmatrix} \cos(180) & -\sin(180) & 0 \\ \sin(180) & \cos(180) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_{AB} = \begin{bmatrix} -1 & 0 & 0 & X_B \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} = T_{BC} \begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} = \begin{bmatrix} R_{AB} & t_{AB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} \qquad t_{AB} = \begin{bmatrix} X_C \\ 0 \\ 0 \end{bmatrix}$$

$$R_{BC} = \begin{bmatrix} \cos(180) & -\sin(180) & 0 \\ \sin(180) & \cos(180) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90) & 0 & \sin(-90) \\ 0 & 1 & 0 \\ -\sin(-90) & 0 & \cos(-90) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(60) & -\sin(60) \\ 0 & \sin(60) & \cos(60) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{bmatrix}$$

$$T_{BC} = \begin{bmatrix} -1 & \frac{\sqrt{3}}{2} & \frac{1}{2} & X_C \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} = T_{CB} \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} = \begin{bmatrix} R_{CB} & t_{CB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \\ 1 \end{bmatrix} \qquad t_{CB} = \begin{bmatrix} 0 \\ 0 \\ Z_B \end{bmatrix}$$

$$R_{CB} = \begin{bmatrix} \cos(-30) & -\sin(-30) & 0 \\ \sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(90) & 0 & \sin(90) \\ 0 & 1 & 0 \\ -\sin(90) & 0 & \cos(90) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(180) & -\sin(180) \\ 0 & \sin(180) & \cos(180) \end{bmatrix}$$

4. (continued)

$$R_{CB} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -1 & 0 & 0 \end{bmatrix}$$

$$T_{CB} = \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -1 & 0 & 0 & Z_B \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.

$$x_1 = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$
$$y_1 = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

$$\frac{\partial x}{\partial \theta_1} = -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)$$

$$\frac{\partial x}{\partial \theta_2} = -l_2 \sin(\theta_1 + \theta_2)$$

$$\frac{\partial y}{\partial \theta_1} = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$

$$\frac{\partial y}{\partial \theta_2} = l_2 \cos(\theta_1 + \theta_2)$$

$$J(\theta_1, \theta_2) = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos\theta_1 - l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

$$\begin{aligned} \det(J) &= -l_1 l_2 \sin\theta_1 \cos(\theta_1+\theta_2) \cancel{- l_2^2 \sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)} + l_1 l_2 \cos\theta_1 \sin(\theta_1+\theta_2) \\ &\quad \cancel{+ l_2^2 \sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)} \\ &= l_1 l_2 \cos\theta_1 \sin(\theta_1+\theta_2) - l_1 l_2 \sin\theta_1 \cos(\theta_1+\theta_2) \end{aligned}$$

$J(\theta_1, \theta_2)$ is singular when $\theta_2 = 0$ and $\theta_1 = 90$ or 0