

Provided for non-commercial research and education use.  
Not for reproduction, distribution or commercial use.



Volumes 460–462, Part 1, 1 September 2007  
Complete volumes

ISSN 0921-4534

**PHYSICA**



Recognized by the European Physical Society

**C**

**SUPERCONDUCTIVITY  
AND ITS APPLICATIONS**



Proceedings of the 8th International Conference  
on Materials and Mechanisms of Superconductivity  
and High Temperature Superconductors

**M2S-HTSC VIII**

Dresden, Germany  
July 9–14, 2006

Part 1

Editors:

Bernd Büchner  
Jörg Fink

Available online at

ScienceDirect  
www.sciencedirect.com

<http://www.elsevier.com/locate/physc>

This article was published in an Elsevier journal. The attached copy is furnished to the author for non-commercial research and education use, including for instruction at the author's institution, sharing with colleagues and providing to institution administration.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



## Unresolved problems in superconductivity of $\text{CaC}_6$

I.I. Mazin <sup>a,\*</sup>, L. Boeri <sup>b</sup>, O.V. Dolgov <sup>b</sup>, A.A. Golubov <sup>c</sup>, G.B. Bachelet <sup>d</sup>,  
M. Giantomassi <sup>e</sup>, O.K. Andersen <sup>b</sup>

<sup>a</sup> Center for Computational Materials Science, Naval Research Laboratory, Washington, DC 20375, USA

<sup>b</sup> Max Planck Institute Fur Festkörperforschung, Stuttgart, Germany

<sup>c</sup> Twente University, Faculty of Science and Technology, Enschede, Netherlands

<sup>d</sup> INFN and Dipartimento di Fisica, La Sapienza, Roma, Italy

<sup>e</sup> Unité PCPM, Université Catholique de Louvain, Louvain-la-Neuve, Belgium

Available online 30 March 2007

---

### Abstract

We discuss the current status of the theory of the “high-temperature” superconductivity in intercalated graphites  $\text{YbC}_6$  and  $\text{CaC}_6$ . We emphasize that while the general picture of conventional, phonon-driven superconductivity has already emerged and is generally accepted, there are still interesting problems with this picture, such as weak coupling regime inferred from specific heat suggesting coupling exclusively with high-energy carbon phonons coming in direct contradiction with the isotope effect measurements in turn suggesting coupling exclusively with the low-energy intercalant modes. At the same time, the first principle calculations, while explaining  $T_c$ , contradict both the experiments above by predicting *equal* coupling with both groups of phonons.

© 2007 Elsevier B.V. All rights reserved.

*Keywords:* Superconductivity; Intercalated graphite

---

### 1. Historical introduction

The fact that graphite, a zero-gap semiconductor, becomes superconducting upon intercalation, has been known for some decades already [1]. However, till recently [2] the critical temperature was not higher than a few Kelvins. Therefore, the discovery of superconductivity in  $\text{YbC}_6$  at 6.5 K, and, a year later, in  $\text{CaC}_6$  [2] at 11.5 K, came as a substantial surprise.

Initially, the fact that Yb often exhibits intermediate valent states inspired speculations of a superconductivity mediated by valence fluctuations. It was found, however [3], by both experimental and theoretical investigations that Yb remains divalent in this compound and its f-electrons play little role in the electronic structure at the Fermi level. Upon the discovery of superconductivity in  $\text{CaC}_6$  Csanyi et al. [4], comparing band structures of supercon-

ducting and non-superconducting intercalated graphites, discovered a correlation between the occupancy of the so-called interlayer band and the appearance (and, to some extent, critical temperature) of superconductivity.

This undisputable correlation led Csanyi et al. to a logical conjecture that this band (its character and formation will be discussed below) must play some important role in superconductivity. However, their next conjecture, that this band does not couple to phonons, was not really based on any computational or experimental fact and turned out to be incorrect [5–7]. This fact to a large extent renders moot the discussion initiated by the exciting suggestion [4] that the superconductivity in  $\text{AC}_6$  ( $A = \text{Yb}, \text{Ca}$ ) is of electronic origin, but it is worth noting that there are even more general reasons to believe that neither of the two proposed electronic mechanisms, Ginzburg’s “sandwich” superconductivity and 2D acoustic plasmon mechanism, can be operative in these compounds. We will discuss this briefly later in the paper and refer the reader for details to Ref. [5].

---

\* Corresponding author. Tel.: +1 202 767 6990; fax: +1 202 404 7546.  
E-mail address: [mazin@nrl.navy.mil](mailto:mazin@nrl.navy.mil) (I.I. Mazin).

The next step in unraveling the mechanism of superconductivity in  $AC_6$  was taken by one of us in Ref. [5], where it was pointed out that the square root of the mass ratio of the intercalants, Yb and Ca, is just 15% larger than the ratio of the critical temperatures. At that moment it had been reported that the crystal structure of the two compounds was identical. This, together with the established valency 2 of Yb in  $YbC_6$ , led to the conclusion [5] that the electronic structures of the two compounds are extremely similar, and therefore the difference in their critical temperatures must be mostly due to the difference in the phonon frequency of the intercalant atoms (which, incidentally, was in line with the trend discovered by Csanyi et al. that filling of the interlayer band correlates with superconductivity). Thus, a prediction was made that the isotope effect on Ca should not be much reduced from its ideal value of 0.5 (15% reduction being a natural rule-of-thumb guess).

However, Emery et al. turned the next page in the story of  $CaC_6$  [9] by showing that its crystal structure is actually different from that of  $YbC_6$ , not much but enough to render Mazin's estimate [5] inaccurate. Indeed, the first principle calculations of Mauri and Calandra [6] and of Boeri et al. [7] showed that the intercalant modes do not *dominate* the electron–phonon coupling, but contribute to it about as much as the graphite modes.

These calculations predicted a moderately strong coupling ( $\lambda \approx 0.85$ ). Solving the Eliashberg equations with the calculated  $\alpha^2F(\omega)$ , with a reasonable  $\mu = 0.12$ , gives the experimental  $T_c$ . While the conclusion that superconductivity in  $AC_6$  is conventional, s-wave, phonon-mediated, has found full support in numerous experiments, as discussed below, there are still alarming quantitative discrepancies between the theory and the experiment, as well as among different experiments.

## 2. Electrons and phonons in $CaC_6$

As discussed above, the main feature that distinguishes the superconducting intercalated graphites is the appearance at the Fermi level of a three-dimensional nearly-free-electron band (NFEB) that some authors (e.g., Ref. [4]) call a free electron band, and others (e.g., Ref. [6]) an intercalant band, yet others an interlayer band (e.g., [8] and references therein). The former refer to the fact that one can remove Ca from the system and still identify the NFEB, but much higher in energy than in  $CaC_6$ . The latter argue that one can remove *carbon* instead, leaving a somewhat expanded along 111 metallic Ca and still see the same band! Of course, the root of the controversy is in the fact that pure Ca is a nearly-free-electron metal and its sp bands are rather similar to the free electron bands (Figs. 1 and 2).

From Fig. 1 one can see that the NFEB is present in both cases. From Fig. 2 it is obvious that there are nevertheless important differences: in  $CaC_6$ , calcium  $d(z^2 - 1)$  orbitals participate in the formation of this band, as well as carbon  $p(z)$ . Most importantly, Fig. 2 clearly shows that

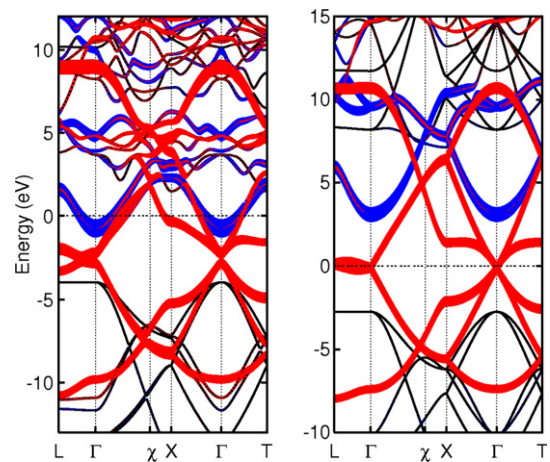


Fig. 1. Left panel: Band structure of  $CaC_6$ , with the NFEB emphasized in blue, and carbon  $\pi$  bands in red. Right panel: the same for a non-intercalated graphite. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of the article.)

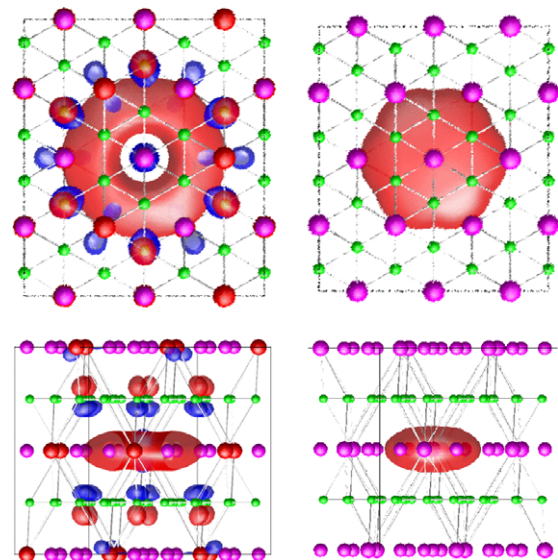


Fig. 2. Isocontours of the amplitude of the Wannier function (red: positive, blue: negative) corresponding to the NFEB. Left panel:  $CaC_6$ , Right panel:  $C_6$ . (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of the article.)

the NFEB electrons are localized in the intercalant plane (even without an intercalant!), and therefore the NFEB should rather be sensible to any charge redistribution in this plane (think intercalant phonons), which it indeed is, as was first pointed out in Ref. [5] and confirmed in Refs. [6,7,15].

Actual linear response calculations [6,7] have produced an interesting and consistent picture: there are three distinctive groups of modes, one at  $\omega \approx 10$  meV, another at  $\omega \approx 60$  meV, and the third at  $\omega \approx 170$  meV, contributing, respectively,  $\approx 0.4$ ,  $\approx 0.35$ , and  $\approx 0.10$  to the total coupling constant  $\lambda$  (Fig. 3). These three groups are mainly composed of the out-of-plane Ca and in-plane C

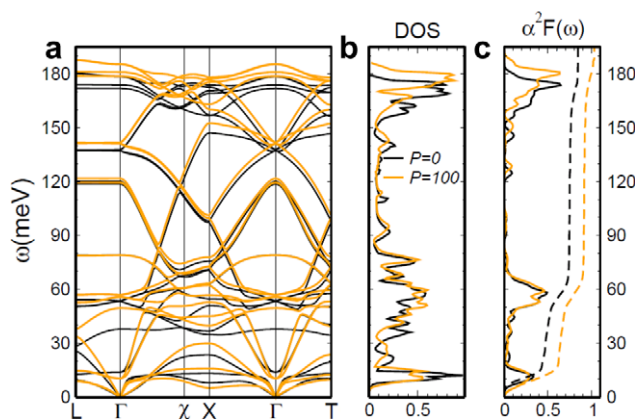


Fig. 3. (a) Calculated phonon dispersion and (b) density of states of  $\text{CaC}_6$  for  $P=0$  and  $P=100$  kbar, (c) Eliashberg function and frequency-dependent electron–phonon coupling constant  $\lambda(\omega) = 2 \int_{x<\omega} \alpha^2 F(x) dx/x$ , from Ref. [7].

vibrations, respectively. Note that the lowest group has the frequency, in Kelvin units, of 100–120 K, while the energy of the most efficient pairing phonons is [10]  $2\pi T_c = 72$  K. Thus these phonons are quite close to the borderline of the applicability of the Eliashberg equations, as discussed in regards to ultrasoft modes in Ref. [11].

Fig. 4 shows the calculated Fermi surface sheets of  $\text{CaC}_6$ . These cannot be separated into a Fermi surface associated with the NFEF and another with the carbon bands, although it is visually clear that some parts of each Fermi surface originate from one set of bands and some others from the other. It is also clear from the figure that on average the electronic structure is fairly isotropic, thanks to the NFEF. Indeed, for  $\text{YbC}_6$  the calculated plasma frequencies [5] are  $\omega_{ab} = 6.2$  eV and  $\omega_c = 3.6$  eV, and for  $\text{CaC}_6$  we have computed in the same way  $\omega_{ab} = 6.6$  eV and  $\omega_c = 3.5$  eV (the corresponding average Fermi velocities are  $5.3 \times 10^7$  and  $2.7 \times 10^8$  cm/s). Our final comment in this section is that up to now the coupling with the Ca phonons has been

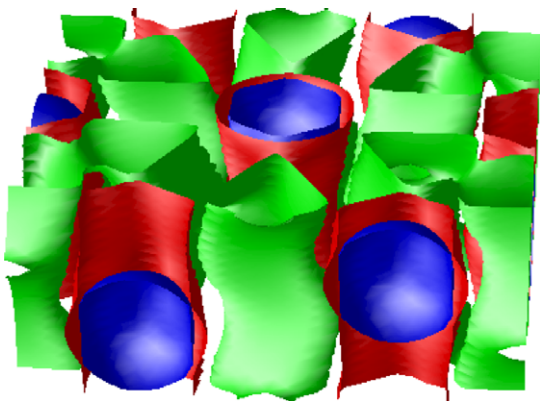


Fig. 4. Fermi surface of  $\text{CaC}_6$ . Note that NFEF contributes to two different Fermi surfaces, blue and red. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of the article.)

discussed only in terms of the linear harmonic response. Softness of the Ca in-plane modes and the very fact that electronically similar  $\text{CaC}_6$  and  $\text{YbC}_6$  assume different stacking sequences for the intercalant layers suggest that anharmonic effects, as well as nonlinear coupling (two-phonon exchange) may be operative in this system, as they possibly are in  $\text{MgB}_2$  [12].

Next we briefly discuss the various models that have been proposed in connection with superconductivity in  $\text{AC}_6$ .

### 2.1. Ginzburg's sandwich

This model calls for a layered structure that consists of highly-conductive (“metallic”) layers and highly polarizable (“semiconducting”) layers. The idea is that the “metallic” electrons polarize the “semiconducting” layer and this dynamic polarization creates a net attractive interaction.

The key elements of this model are high polarizability and small characteristic energy for the “semiconducting” layer. The latter condition is necessary for suppressing the direct Coulomb repulsion via the Tolmachev logarithm. Unfortunately, exactly in the superconducting graphites the NFEF crosses the Fermi level and renders both carbon and intercalant layers metallic thus excluding the possibility of “sandwich” superconductivity.

### 2.2. Acoustic plasmons

Just as the previous model, the idea of superconductivity due to exchange of acoustic plasmons is about 40 years old, and not dissimilar to the former. In this model, electronic systems that provide the carrier and polarization are intertwined. The same conditions apply: high polarizability, and low characteristic frequency, and one other condition appears: that of stability against a charge density wave formation. Note that the criterion for an attractive pairing interaction in the plasmon model differs from that of a CDW instability only by vertex corrections, which usually severely restricts the admissible parameter space in model calculations.

Two types of acoustic plasmons have been discussed in the literature. One refers to the Goldstone mode that appears in any system with charge carriers of vastly different masses (cf. acoustic phonons). This model is clearly inapplicable to  $\text{AC}_6$ , where only a very moderate electronic mass variation exists. The other model applies to a strongly 2D system, where the Fermi velocity in one direction is much smaller than in the others. Since the plasma frequency can be calculated from the latter and the density of states  $N(0)$  as  $m\omega_p^2 = 4\pi \langle N v_F^2 \rangle_{\text{FS}}$ , a small Fermi velocity in one direction may lead to a very small plasma frequency in the same directions and in principle one may think about superconductivity mediated by such plasmons. Unfortunately, as mentioned above, the partial filling of the NFEF ensures that the plasma frequency along the  $c$  direction is

of the order of several eV, i.e., 2D acoustic phonons are not present in the system.

### 2.3. Multiband superconductivity

Although currently the interest to multigap superconductivity is high, thanks to  $\text{MgB}_2$ , the idea of a two-gap superconductivity in intercalated graphites is very old [13]. The problem here is that, as Fig. 4 shows, in this case one can speak, at best, about anisotropic superconductivity with an order parameter radically changing over one and the same Fermi surface sheet. While not impossible, it makes a two-gap superconductivity less likely. Indeed, experimental evidence is mounting in favor of one isotropic order parameter [14–17].

## 3. Unsolved problems

At first glance, it may seem that the superconductivity in  $\text{AC}_6$  is fully explained in terms of the conventional, standard, isotropic BCS theory. Unfortunately, this is not the case. Below we list the questions that still await answers.

### 3.1. Critical field

As pointed out in Refs. [15,18],  $H_{c2}(T)$  remains linear down to 1 K, which contradicts the standard BCS model. In principle, a very strong coupling can render a linear  $H_{c2}(T)$  [11]; however, we have calculated  $H_{c2}(T)$  using the LDA  $\alpha^2F(\omega)$  and found that sublinear behavior starts already at  $T < 4$  K. A more likely explanation has to do with the Fermi velocity anisotropy, which can lead to a linear  $H_{c2}$  even in a cubic superconductor like  $\text{K}_3\text{C}_{60}$  [19]. A good rule-of-thumb is that if half of the Fermi surface has a velocity twice larger than the other half,  $H_{c2}(T)$  is linear [e.g., according to the cubic formula  $H_{c2}(0)/H_{c2}^{\text{BCS}}(0) = \langle v_F^2 \rangle / \langle \exp(\ln v_F^2) \rangle$ ]. However, this explanation needs a quantitative verification.

### 3.2. Specific heat

An analysis of  $C(T)$  below  $T_c$  in the framework of the  $\alpha$ -model [15] yields  $\alpha = 1.776$ , within 1% of the BCS value.  $C/T$  renormalization is 1.6, to be compared to the calculated [6,7]  $\lambda = 0.85$ . This seems like a good agreement, but, as Fig. 5 shows, the calculated  $\alpha^2F(\omega)$  produces strong coupling effects in  $C(T)$  well beyond the experimental error. We have also checked that the disagreement is stable against the shape of  $\alpha^2F(\omega)$ , by repeating the calculations using the same  $\lambda = 0.85$ , but condensing the  $\alpha^2F(\omega)$  to one Einstein mode. The shape of  $\Delta C(T)/T$  did change, but not enough to explain the experiment.

### 3.3. Isotope effect

Hinks et al. [20] have measured the isotope effect on Ca and found it to be 0.5, as opposed to 0.25 predicted by the

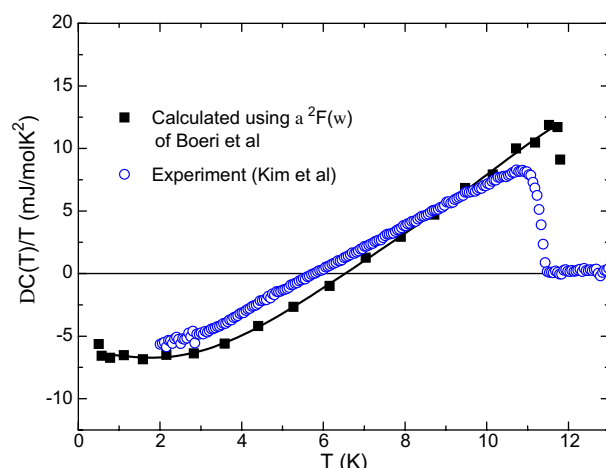


Fig. 5. Deviation of the calculated (strong coupling) specific heat from the experiment [15].

calculations [6]. Since all Ca modes are rather soft, coupling exclusively with Ca phonons must necessarily be very strong, in disagreement with the specific heat data suggesting a typical weak coupling regime, and with other indications of weak coupling.

Thus, the main challenge regarding the superconductivity in  $\text{AC}_6$  is to reconcile the evidence for weak coupling (i.e., dominance of high-energy C modes) [15] with evidence for the dominance of the intercalant modes (i.e., strong coupling) [20] and with the computations [6,7] and experiment [21] (this experiment showed that both types of modes are needed to quantitatively explain the temperature dependence of resistivity, and that with pressure the coupling with all modes remains constant, while the Ca modes soften until becoming unstable) predicting intermediate coupling with both C and Ca equally involved.

### Note added

After this paper had been submitted, density functional calculations suggested a moderately anisotropic order parameter have been published [22] that may be capable of resolving the specific heat problem. The large Ca isotope effect still remains a mystery, though.

## References

- [1] I.T. Belash, O.V. Zharikov, A.V. Palmichenko, Synt. Metals 34 (1989) 455; I.T. Belash, O.V. Zharikov, A.V. Palmichenko, Synt. Metals 36 (302) (1989) 283.
- [2] T.E. Weller, M. Ellerby, S.S. Saxena, R.P. Smith, N.T. Skipper, Nature Physics 1 (2005) 39.
- [3] I.I. Mazin, S.L. Molodtsov, Phys. Rev. B 72 (2005) 172504.
- [4] G. Csanyi, P.B. Littlewood, A.H. Nevidomskyy, C.J. Pickard, B.D. Simons, Nature Physics 1 (2005) 42.
- [5] I.I. Mazin, Phys. Rev. Lett. 95 (2005) 227001.
- [6] M. Calandra, F. Mauri, Phys. Rev. Lett. 95 (2005) 237002.
- [7] J.S. Kim, L. Boeri, R.K. Kremer, F.S. Razavi, Phys. Rev. 95 (2006) 214513.

- [8] M. Giantomassi, L. Boeri, G.B. Bachelet, *Phys. Rev. B* 72 (2006) 224512.
- [9] N. Emery, C. Herold, P. Lagrange, *J. Solid State Chem.* 178 (2005) 2947;  
N. Emery, C. Hérold, M. d'Astuto, V. Garcia, Ch. Bellin, J.F. Maréché, P. Lagrange, G. Loupiau, *Phys. Rev. Lett.* 95 (2005) 087003.
- [10] J. Carbotte, *Rev. Mod. Phys.* 62 (1990) 1027.
- [11] I.I. Mazin, O.V. Dolgov, A. Golubov, S.V. Shulga, *Phys. Rev. B* 47 (1993) 538.
- [12] I.I. Mazin, V.P. Antropov, *Physica C* 385 (2003) 49.
- [13] R. Al-Jishi, *Phys. Rev. B* 28 (1983) 112.
- [14] G. Lamura, M. Aurino, G. Cifariello, E. Di Gennaro, A. Andreone, N. Emery, C. Hérold, J.-F. Maréché, P. Lagrange, *Phys. Rev. Lett.* 96 (2006) 107008.
- [15] J.S. Kim, R.K. Kremer, L. Boeri, F.S. Razavi, *Phys. Rev. Lett.* 96 (2006) 217002.
- [16] N. Bergeal, V. Dubost, Y. Noat, W. Sacks, D. Roditchev, N. Emery, C. Herold, J.-F. Mareche, P. Lagrange, G. Loupiau, *Phys. Rev. Lett.* 97 (2006) 077003.
- [17] M. Sutherland, N. Doiron-Leyraud, L. Taillefer, T. Weller, M. Ellerby, S.S. Saxena, *Phys. Rev. Lett.* 98 (2007) 067003.
- [18] E. Joubinong, H.D. Zhou, J.A. Janik, et al., *cond-mat/0604062*.
- [19] G.S. Boebinger, T.T.M. Palstra, A. Passner, M.J. Rosseinsky, D.W. Murphy, I.I. Mazin, *Phys. Rev. B* 46 (1992) 5876.
- [20] D.G. Hinks, D. Rosenmann, H. Claus, M.S. Bailey, J.D. Jorgensen, *Phys. Rev. B* 75 (2007) 014509.
- [21] A. Gauzzi, S. Takashima, N. Takeshita, C. Terakura, H. Takagi, N. Emery, C. Herold, P. Lagrange, G. Loupiau, *Phys. Rev. Lett.* 98 (2007) 067002.
- [22] A. Sanna, G. Profeta, A. Floris, A. Marini, E.K.U. Gross, S. Massidda, *Phys. Rev. B* 75 (2007) 020511(R).