Notes on altermagnetism and superconductivity

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Altermagnetism is a recently discovered new type of collinear magnets, which share some characteristic features with ferromagnets (lack of the nonrelativistic Kramers degeneracy at a general point in the Brillouin zone, finite anomalous Hall effect, finite magnetooptical effect) and other with antiferromagnets (net magnetization zero by symmetry)[1]. While numerous properties of altermagnets have been explored, largely from the point of view of spintronics, interplay between superconductivity and altermagnetism, another aspect in which ferro- and antiferromagnets are principally different, has not been addressed so far. Not surprisingly, there altermagnets can can manifest properties typical for ferromagnets in one contexts, and those typical for antiferromagnets in another.

There are two issues that are typically considered in terms of interaction between magnetism and superconductivity: (1) what kind of superconducting state may be consistent with a given magnetic order and (2) what kind of pairing can be generated by proximity to a magnetic order (in other words, if we can gradually suppress the long range magnetic order by an external stimulus, such as pressure, what supperconducting symmetry may emerge on the either side of the quantum crutical point?).

Superconductivity and ordered altermagnetism It is well known that the standard antiferromagnetism can support singlet superconductivity (as for instance in Fe-based superconductors) as long as the coherence length is much larger than the period of antiferromagnetic order. On the other hand, a split ferromagnet with spin-split bands (*i.e.*, the eigenvalues $\epsilon_{\mathbf{k}\uparrow} \neq \epsilon_{-\mathbf{k}\downarrow}$) can only support Cooper pairs with the spinor order parameter $\Delta_{\uparrow\uparrow}$, which is triplet. A standard representation[2] of the spinor triplet order parameter in terms of a spacial vector $\mathbf{d}$ describe this spinor as

$$\Delta_{\alpha\beta} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix} \tag{1}$$

Obviously, since only the $\alpha = \beta =\uparrow$ element is nonzero for a given $\mathbf{k}$, $d_z = 0$, and $d_x = -id_y$.

AM is rather close to FM in this sense, but it has an additional symmetry: there is an element of the point group that does not map the Fermi surface for a given spin upon itself, but does map it upon the Fermi surface for the opposite spin[1, 6]. Let us now, for simplicity, consider a tetragonal FM material. Then the only triplet state consistent with this requirement is the nonunitary state $\mathbf{d}= F_1(k)k_z(\hat{x} + i\hat{y})$, where F has the full tetragonal symmetry, so that of $\Delta_{\alpha\beta}$ only $\Delta_{\uparrow\uparrow} \neq 0$. If we now consider the other Fermi surface, for the opposite spin, the order parameter there will be $\mathbf{d} =F_2(k)k_z(\hat{x} - i\hat{y})$, with only $\Delta_{\downarrow\downarrow} \neq 0$. These two order parameters have different symmetries, and are not degenerate, so the critical temperature T_c will be different. The symmetry consideration dictate that, without spin-orbit interaction, there will be no coupling between the two order parameters. In a typical experiment probing the average order parameter such a system at low temperature will behave as mixed state $\mathbf{d} =\frac{k_z}{2}[F_1(k)+F_2(k)]\hat{x}+i\frac{k_z}{2}[F_1(k)-F_2(k)]\hat{y}$. This state is *nonunitary* as long as $F_1 \neq F_2$, and nematic (breaks the C_4 symmetry).

AM, despite the absence of the net magnetization, behaves very much like a ferromagnet in the sense that any Cooper paper can be either $\Delta_{\uparrow\uparrow}$ or $\Delta_{\downarrow\downarrow}$. Correspondingly, the order parameters will be $F_1(k)k_z(\hat{x}+i\hat{y})$ and $F_2(k)k_z(\hat{x} - i\hat{y})$. However, in this case $F_1 = F_2$ by symmetry (the same symmetry that transfoms one spin sublattice into the other), so the average order parameter will be just $\mathbf{d} =k_zF(k)\hat{x}$. This order parameter is strictly *unitary* (correspondingly, the condensate is not spin-polarized, just as the normal state isn't), and nematic. Of course, the partner state $\mathbf{d} =k_zF(k)\hat{y}$ will be degenerate with this one. In this sense, the AM as regards superconductivity again has some features similar to ferromagnets, some similar to antiferromagnets, and some unique. An interesting analogy may be drawn with the Ising superconductivity, appearing when the Kramers degeneracy is lifted not by the exchange field, but by the spin-orbit coupling. In that case the two spin-split Fermi surfaces carry order parameters that are strictly $S + T$ and $S - T$, where $S(T)$ stands for singlet(triplet)[7]. Despite that, in most experiments, namely those that probe the average order parameters, they behave *approximately* as singlet (approximately because no symmetry requires the two order parameters to be exactly the same). In case of AM the difference is that the average order parameter becomes unitary *exactly*, by symmetry (of course, remaining triplet)

Superconductivity and altermagnetic fluctuations A related question is what superconducting symmetry can be generated by the AM-type spin fluctuations. Before discussing that let us compare FM and AF fluctuations a bit more carefully than how it is usually done.

In case of ferromagnetic fluctuations, the spin fluctuation spectrum is peaked at $\mathbf{q} = 0$. Given that spin fluctuations are repulsive in the singlet channel (the partners in a Cooper pair interact to a spin fluctuations with opposite signs), and that by continuity $\Delta(\mathbf{k}) \approx\Delta(\mathbf{k} + \mathbf{q})$, as long as $\mathbf{q}$ is small, such fluctuations will always be pair-breaking. Traditional, Néel type AF order at a finite vector $\mathbf{q}$ (if $\mathbf{q}$ lies at the zone boundary, this order will correspond to doubling of the unit cell). In that case spin fluctuations can be pairing as long as $\Delta(\mathbf{k})\cdot\Delta(\mathbf{k}+\mathbf{q}) < 0$. Popular theories ascribing the d-wave superconductivity in cuprates ($\mathbf{q}_{2D} = \{\pi, \pi\}$) and the $s_\pm$ superconductivity in Fe pnictides ($\mathbf{q}_{2D} = \{\pi, 0\}$) to spin fluctuations utilize this property. While such fluctuations can also generate a p-wave pairing, especially when combined with an anisotropic electron-phonon coupling[3], in practice it is very difficult[4].

In such discussions it is routinely assumed that AF fluctuations *always* correspond to a finite $\mathbf{q}$. However, several hundreds of known antiferromagnets have magnetic order corresponding to $\mathbf{q} = \mathbf{0}$. This can happen, of course, if the magnetic species occupies a Wyckoff position with a multiplicity larger that one. For instance, the very popular now family of Kagome superconductors, $A\mathrm{V_3Sb_5}$ (A is an alkaline metal) is believed to host spin-fluctuations at $\mathbf{q} \approx \mathbf{0}$ and the intra-triangular correlations of 120°. Of course, magnetic order and spin fluctuations $\mathbf{q} \approx \mathbf{0}$ and collinear spins are also perfectly possible, and altermagnetic (and some conventional antiferromagnets) belong to this class.

In order to understand the physics of $\mathbf{q} \approx \mathbf{0}$ spin fluctuations in the context of superconductivity, let us consider a hypothetical 2D lattice depicted in Fig. 1(left). Here M is a metal ion and L is a ligand. This is a tetragonal structure with the symmetry group I4/mmm. Lets us assume that this structure generates a Fermi surface centered around the X(Y) points, as shown in Fig. 2 (left), and spin-fluctuations corresponding to $\mathbf{q} =\{\pi, \pi\}$.This model was introduced by Agterberg *et al*[8], and it leads a d-wave superconductivity of the type $k_x^2 - k_y^2$.

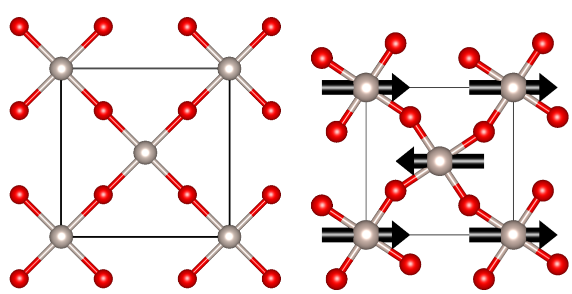

FIG. 1. (left) An example of a 2D P4/mmm structure. The grey balls are metallic (M), and potentially magnetic ions, and the red ones are ligands (L). (right) Same for a P4/mbm structure, which can carry an altermagnetic state (shown by arrows)

Let us now introduce a small distortion, rotations of the ML_2 squares, shown in Fig. 1(right). The symmetry group is now P4/mbm, it is also tetragonal, but has now two metal ions per cell. This will lead to folding down of the original Brillouin zone (Fig. 2), so that now there are to Fermi contours around each M point; if the original Fermi contours were close to circular, the downfolded one will be nearly degenerate.

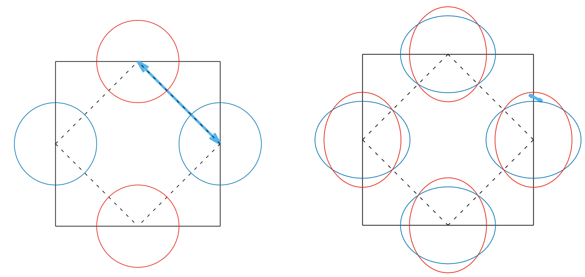

FIG. 2. (left) A possible 2D Brillouin zone consistent with the crystal symmetry shown in Fig. 1(left). (right) The same, after downfolding corresponding to the double unit cell shown in Fig. 1(right). The downfolded zone is shown with the dashed lines. Colors reflect the signs of a possible d-wave (in the unfolded zone) order parameter. The arrows give examples of antiferromagnetic spin fluctuations with $\mathbf{q} \approx \pi, \pi$ generating a d-wave pairing. Nothe that the case on the right these are fromally altermagnetic fluctuations.

Altermagnetic spin fluctuations will have $\mathbf{q} \approx \mathbf{0}$, however, this does not mean that, as in for ferromagnetic fluctuations, such fluctuations can only generate triplet pairing. In fact, since in this particular example the AM order only slightly deviates from the AF order, the generated pairing state must be close to the downfolded d-state of Ref. [8]. In principle, the two crossing Fermi lines will hybridize, and the order parameter form nodal lines, as discussed in Ref. [9]. However, this is a relatively unimporant effect.

From the formal point of view, the issue is that $\mathbf{q} \approx \mathbf{0}$ the spin susceptibility may have importand internal structure, and has to be written as $\chi(\mathbf{q}, \mathbf{r}_1, \mathbf{r}_2)$, where $\mathbf{r}$ is defined inside the first unit cell, or as a matrix in reciprocal vectors, $\chi(\mathbf{q}+\mathbf{G}, \mathbf{q}+\mathbf{G}')$. The corresponding vertex will be determined by the variation of the (non-magnetic) one-electron Green function with respect to a fluctuation generating opposite magnetic moments on the two sublattices. A more detailed theory than that usually used for ferro- or antiferromagnetic spin fluctuations needs to be developed, and it is not unreasonable to assume that both triplet and singlet pairing can be iduced by AM spin fluctuations, depending on the details.

Lastly, one can make another interesting observation: so far, two very different classes of superconductors offer protection from thermodynamic pair breaking (Pauli limiting) for some directions of magnetic fields. The first one are triplet superconductors without net magnetization (such as ^{3}He or the initial (now debunked) model for $\mathrm{Sr_2RuO_4}$); there the spin susceptibility in the superconducting state is the same as in the normal state, $\chi_{sc} = \chi_n$, due to triplet pairs having the same ability to screen the field (in some directions) as the individual electrons. Unitary triplet superconductivity, often discussed in connection with some ferromagnetic U compounds, is also protected, and again $\chi_{sc} = \chi_n$, but there the underlying mechanism is different: If there is and easy-axis magnetocrystalline anisotropy in the normal state, then screening of a small external field is afforder not by increasing the net number of electrons in one spin channel at the expense of the other, but by canting spins of electrons removed from the Fermi level. Indeed, if an external field H is applied perpendicular to this axis, the Fermi surface does not change in the linear in H order, but a linear in H magnetization does appear, on the order of $H\Delta\Omega/\langle\delta V_{xc}\rangle$, where $\Delta\Omega$ is the volume difference between the two spin-split Fermi surfaces and $\langle\delta V_{xc}\rangle$ is the properly averaged exchange splitting. Obviously, the ratio $\Delta\Omega/\langle\delta V_{xc}\rangle$ is a number on the order of the density of states, so this provides a contribution to susceptibilty on the order of the Pauli susceptibility, and is not affected by opening a superconducting gap $\Delta \ll \langle\delta V_{xc}\rangle$. Note that similar protections is operative in Ising superconductors, where the role of δV_{xc} is played by the spin-orbit coupling[7]. In the spirit of the key feature of AM, namely sharing features of both FM and AFM, they also have a Pauli protection, but which in this case is similar to that in ferromagnets, so it does not require accounting for relativistic effects in the band structure, but requires a magnetic anisotropy.

In the above discussion, we have addressed issues related to possible coexistance of AM and superconductivity, as well as superconductivity possibly induced by AM spin fluctuation. A further step in investigating the interplay between magnetism and superconductivity would involve possible effects at the *interface* between a conventional superconductor and an altermagnet.

One of the most interesting effects in this environment is spin-polarized Andreev reflection. Andreev reflection at a boundary between a conventional superconductor and a ferromagnet is well understood[10] and is often use to measure the transport spin polarization of ferromagnets. In a generic ferromagnet, as opposed to a traditional antiferromagnet, the number of conductivity channels for two spins are not the same (in other words, the area of the Fermi surface projection onto the interface is spin-dependent. Since an Andreev process consist of a spin-up electron with a momentum $\mathbf{k}$ and a spin-down one with a momentum $-\mathbf{k}$, some electrons will never find a partner and therefore the conventional Andreev conductivity, which is twice the normal conductivity, will be suppressed. So defined spin polarization depends on the orientation of the interface. While for a ferromagnet is can only be zero by accident, in an AM, for particular interface orientations the number of conductivity channels is the same for both spins, therefore one expects no suppression, just as in an AF, but in some other, and in fact in general directions the areas of the two projections will be different, and a finite suppression will be measured. Again, an AM sometimes behaves as an AF, and sometimes as a FM. This is illustrated in Fig. 3(left), where a single pocket of the Fermi surface of the hypothetical AM $FeSb_2$[6] at a particular Fermi energy is cut off to

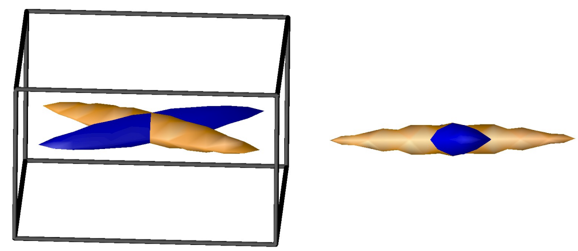

FIG. 3. (left) A possible 2D Brillouin zone consistent with the crystal symmetry shown in Fig. 1(left). Specifically, a cut-off of a specific Fermi surface pocket in the hypothetical altermagnetic $FeSb_2$**??**. Different colors denote different spins. (right) Projection of this pocket onto the (110) inteface.

show the symmetry. Evidently, for a {100), or (010), or (001) interface every $\mathbf{k}_{||}$ has a partner with $-\mathbf{k}_{||}$ and the opposite spin, for the (110) interface, for instance, this is not the case, as is quite obvious from Fig.3(right), where the projections of the two Fermi surfaces are shown.

In this note we have analyized various aspects of interplay between the novel magnetic phenomenon, altermagnetism, and superconductivity. This analysis should be helpful in designing new experiments to further study this unusual state.

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