(1) $x^2+y^2+z^2+A_1^2+B_1^2+C_1^2-2(A_1x+B_1y+C_1z)=c^2(t_1^2+d^2-2t_1d)$

(2) $x^2+y^2+z^2+A_2^2+B_2^2+C_2^2-2(A_2x+B_2y+C_2z)=c^2(t_2^2+d^2-2t_2d)$

(3) $\vdots$

(4)

(1) − (2) : $A_1^2-A_2^2+B_1^2-B_2^2+C_1^2-C_2^2+2x(A_2-A_1)+2y(B_2-B_1)+2z(C_2-C_1)=$
$$c^2(t_1^2-t_2^2+2d(t_2-t_1))$$

(1) − (3) : $A_1^2-A_3^2+B_1^2-B_3^2+C_1^2-C_3^2+2x(A_3-A_1)+2y(B_3-B_1)+2z(C_3-C_1)=$
$$c^2(t_1^2-t_3^2+2d(t_3-t_1))$$

(1) − (4) : $A_1^2-A_4^2+B_1^2-B_4^2+C_1^2-C_4^2+2x(A_4-A_1)+2y(B_4-B_1)+2z(C_4-C_1)=$
$$c^2(t_1^2-t_4^2+2d(t_4-t_1))$$

$$\begin{array}{ccc} \vec{u_x} & \vec{u_y} & \vec{u_z} \\ \end{array}$$
$$\begin{bmatrix} 2(A_2-A_1) & 2(B_2-B_1) & 2(C_2-C_1) \\ 2(A_3-A_1) & 2(B_3-B_1) & 2(C_3-C_1) \\ 2(A_4-A_1) & 2(B_4-B_1) & 2(C_4-C_1) \end{bmatrix}$$

$$\begin{array}{cc} 4^{th}\ \vec{u_d} & 5^{th}\ \vec{w} \end{array}$$
$$\begin{bmatrix} 2c^2(t_1-t_2) & A_1^2-A_2^2+B_1^2-B_2^2+C_1^2-C_2^2+c^2t_2^2-c^2t_1^2 \\ 2c^2(t_1-t_3) & A_1^2-A_3^2+B_1^2-B_3^2+C_1^2-C_3^2+c^2t_3^2-c^2t_1^2 \\ 2c^2(t_1-t_4) & A_1^2-A_4^2+B_1^2-B_4^2+C_1^2-C_4^2+c^2t_4^2-c^2t_1^2 \end{bmatrix}$$

$$0 = \det\left(\vec{u_y} \,\middle|\, \vec{u_z} \,\middle|\, x\vec{u_x} + yu_y + zu_{\vec{z}} + d\vec{u_d} + \vec{w}\right)$$

$$0 = x \underbrace{\det\left(\vec{u_y} \,|\, \vec{u_z} \,|\, \vec{u_x}\right)}_{l_1} + d \cdot \underbrace{\det\left(\vec{u_y} \,|\, \vec{u_z} \,|\, \vec{u_d}\right)}_{l_2} + \underbrace{\det\left(\vec{u_y} \,|\, \vec{u_z} \,|\, \vec{w}\right)}_{l_3}$$

$$0 = x\,l_1 + d\,l_2 + l_3 \quad \therefore \quad x = \frac{-d\,l_2 - l_3}{l_1}$$

Y component:

$$0 = \det\left(u_x \,\middle|\, u_z \,\middle|\, x\vec{u_x} + y\vec{u_y} + z\vec{u_z} + d\vec{u_d} + \vec{w}\right)$$

$$0 = y \underbrace{\det\left(u_x \,|\, u_z \,|\, u_y\right)}_{m_1} + d \cdot \underbrace{\det\left(\vec{u_x} \,|\, \vec{u_z} \,|\, \vec{u_d}\right)}_{m_2} + \underbrace{\det\left(\vec{u_x} \,|\, \vec{u_z} \,|\, \vec{w}\right)}_{m_3}$$

$$y = \frac{-d\,m_2 - m_3}{m_1}$$

Z-comp

$$0 = \det\left(u_x \,\middle|\, u_y \,\middle|\, x\vec{u_x} + y\vec{u_y} + z\vec{u_z} + d\vec{u_d} + \vec{w}\right)$$

$$0 = z \underbrace{\det\left(\vec{u_x} \,|\, \vec{u_y} \,|\, \vec{u_z}\right)}_{n_1} + d \cdot \underbrace{\det\left(\vec{u_x} \,|\, \vec{u_y} \,|\, \vec{u_d}\right)}_{n_2} + \underbrace{\det\left(\vec{u_x} \,|\, \vec{u_y} \,|\, \vec{w}\right)}_{n_3}$$

$$z = \frac{-d\,n_2 - n_3}{n_1}$$

$e_1 = \frac{-l_2}{l_1}$, $e_2 = \frac{-l_3}{l_1}$, $e_3 = \frac{-m_2}{m_1}$, $e_4 = \frac{-m_3}{m_1}$

$e_5 = \frac{-n_2}{n_1}$, $e_6 = \frac{-n_3}{n_1}$

$$x = \frac{-d l_2}{l_1} - \frac{l_3}{l_1} = e_1 d + e_2$$

$$y = \frac{-d m_2}{m_1} - \frac{m_3}{m_1} = e_3 d + e_4$$

$$z = \frac{-d n_2}{n_1} - \frac{n_3}{n_1} = e_5 d + e_6$$

so $x^2 = e_1^2 d^2 + e_2^2 + 2 e_1 e_2 d$

$y^2 = e_3^2 d^2 + e_4^2 + 2 e_3 e_4 d$

$z^2 = e_5^2 d^2 + e_6^2 + 2 e_5 e_6 d$

so eq (1):

$$e_1^2 d^2 + e_3^2 d^2 + e_5 d^2 + 2d(e_1e_2 + e_3e_4 + e_5e_6) + e_2^2 + e_4^2 + e_6^2 + A_1^2 + B_1^2 + C_1^2$$

$$-2(A_1 e_1 d + A_1 e_2 + B_1 e_3 d + B_1 e_4 + C_1 e_5 d + C_1 e_6)$$

$$+ c^2(2t_1 d - t_1^2 - d^2) = 0$$

so

d^2 G

$$d^2(e_1^2 + e_3^2 + e_5^2 - c^2)$$

d H

$$d\left(2 \cdot \left(e_1e_2 + e_3e_4 + e_5e_6 - A_1e_1 - B_1e_3 - C_1e_5 + c^2 t_1\right)\right)$$

d^0 I

$$e_2^2 + e_4^2 + e_6^2 - 2(A_1 e_2 + B_1 e_4 + C_1 e_6) - c^2 t_1^2 + A_1^2 + B_1^2 + C_1^2$$

$$d = \frac{-H \pm \sqrt{H^2 - 4 \cdot G \cdot I}}{2G}$$