Geometry Assessment

Gwenanne Salkind

George Mason University

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Dr. Anthony E. Kelly

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Introduction

“Assessment is a process for obtaining information for making a particular educational decision” (Nitko, 2004, p. 6). In EDIT 746 we talked about many forms of assessment and educational decisions. In order to practice and apply the ideas presented in EDIT 746, I created a geometry assessment for fifth grade students. I do not teach fifth grade but work with elementary mathematics teachers at all grades. This project was useful for my own learning, but I will not have the opportunity to implement the assessment with fifth graders.

If this assessment were used in a fifth grade class, it would help the teacher to make many decisions. She might use the assessment as a pre-test to make placement decisions about how to group children during mathematics instruction. Used in this way, as formative assessment, the teacher would plan instruction that addresses students’ needs as identified on the test. She could use it as summative assessment to assign report card grades. She could use it as talking points to report progress to parents during a parent/teacher conference. Even though the test was designed to measure student knowledge of geometry, the teacher could analyze completed tests to evaluate her own teaching. For example, if the majority of the students missed the same problem, she may think back to how she taught that particular concept and adjust her teaching method. I caution the teacher in using just one test to make important decisions, however. I suggest that the teacher consider multiple assessments when making important decisions such as students’ report card grades, promotion, and retention.

Learning Targets

I used Chapter 2 and Appendices D and E in Educational Assessment of Students (Nitko, 2004) and the Virginia Department of Education Mathematics Standards of Learning Curriculum Framework (2002) to help me define the learning targets for my geometry unit. In
chapter 2, Nitko (2004) explains the differences among educational goals, general learning targets, and specific learning targets. An educational goal is broad. It gives “direction and purpose to planning overall educational activities” (p. 17). A general learning target is derived from the educational goal. It is more specific than the educational goal. The specific learning target is even more specific. I have listed the educational goal, the general learning targets, and the specific learning targets for my geometry unit below. I have also place the complete list in Appendix A for ease of referencing. I will discuss each target briefly.

*Educational Goal: Students will use geometry to represent and make sense of the world.*

This goal is broad. It spans all grade levels. The subject-matter of the goal is specific to mathematics. It gives purpose and direction to planning mathematics activities for students.

*General Learning Targets*

1. Students will use mathematical reasoning to analyze and justify properties and relationships among shapes.

2. Students will visualize, draw, measure, compare, and classify geometric shapes.

These learning targets are general. They are more specific than the educational goal, but they are not specific enough to measure student achievement against them.

*Specific Learning Targets*

1. Students will recognize and name plane geometric figures (triangle, square, rectangle, and circle).

2. Students will identify plane geometric figures (triangle, quadrilateral, pentagon, hexagon, and octagon) according to the number of sides and angles.

3. Students will compare and contrast plane geometric shapes.
4. Students will analyze the properties of quadrilaterals (squares, rectangles, parallelograms, rhombi, kites, and trapezoids).

5. Students will classify quadrilaterals according to their properties.

6. Students will measure and draw right, acute, and obtuse angles.

7. Students will classify triangles as equilateral, scalene, isosceles, right, acute, and obtuse.

8. Students will effectively communicate their mathematical reasoning orally and in writing.

These learning targets specify what a student should know and be able to do after instruction. One danger in creating specific learning targets is that the target could be too specific. Targets that are too specific may be based upon the assessment task instead of the actual learning goal. I believe that the learning targets listed above are specific enough to be measured, but not too specific.

All of the learning targets are cognitive targets. Targets 6 and 7 also have psychomotor aspects along with the cognitive aspects. Seven of the eight targets are mastery targets. Target 8 is a developmental target. Students will continue to develop mathematical communication skills throughout their lives. This target will be assessed based on expectations for typical fifth grade students.

The Learning Targets and the Taxonomies

In this section of the paper, I connect each learning target to taxonomies of educational objectives. Many of the taxonomies I discuss come from Nitko (2004, pp. 460-467). I have added one taxonomy to Nitko’s list, the van Hiele levels of geometric reasoning (see Appendix B). “Two mathematics teachers from the Netherlands, Pierre van Hiele and Dina van Hiele, identified several levels of reasoning that students pass through as they study geometry”
Since my topic is geometry, it is important to link the learning targets to the van Hiele levels. Fifth graders are expected to be functioning at the third level of the taxonomy.

**Target 1:** Students will recognize and name plane geometric figures (triangle, square, rectangle, and circle). This target matches Level 1: Visualization on van Hiele’s taxonomy, the knowledge level of Bloom’s taxonomy, and the recall level of Quellmalz taxonomy. It involves recall of shapes and names of shapes. Names of shapes would be considered declarative knowledge on the Dimensions of Learning taxonomy (Nitko, 2004, p. 25). I would also place this target on Gagne’s discrimination level because the students are discriminating among shapes.

**Target 2:** Students will identify plane geometric figures (triangle, quadrilateral, pentagon, hexagon, and octagon) according to the number of sides and angles. This target fits with van Hiele’s Level 2: Analysis and Gagne’s concrete concept level because students identify the shape based upon the shape’s physical properties. I would put it in Quellmalz recall level and Bloom’s knowledge level. This target is low level on the taxonomies. Students could just memorize what they need to know.

**Target 3:** Students will compare and contrast plane geometric shapes. This target is slightly higher on the taxonomies than Targets 1 and 2. It is on Bloom’s analysis level, Quellmalz comparison level, and Reasoning Strategy 1: Comparison on the Dimensions of Learning Model. Students must analyze and describe similarities and differences between two or more items. I would place this target on van Hiele’s Level 2: Analysis because students compare shapes by identifying the properties of the shapes, but they are not necessarily defining classes and subclasses of shapes.

**Target 4:** Students will analyze the properties of quadrilaterals (squares, rectangles, parallelograms, rhombi, kites, and trapezoids). This target is on van Hiele’s Level 2: Analysis,
Bloom’s *analysis* level, Gagne’s *defined concept* level, and Quellmalz *analysis* level. It is also *Reasoning Strategy 1: Comparison* on the Dimensions of Learning Model because students must compare the different quadrilaterals in order to analyze their properties.

**Target 5:** *Students will classify quadrilaterals according to their properties.* This target falls under the Dimensions of Learning Model *Reasoning Strategy 2: Classification.* It is also Bloom’s *synthesis* level, Gagne’s *defined concepts* level, and van Hiele’s taxonomy, *Level 3: Abstraction.* Students are now finding groups and subgroups of quadrilaterals.

**Target 6:** *Students will measure and draw right, acute, and obtuse angles.* This target requires students to use tools to draw and measure. Therefore, it involves psychomotor skills at the *perceptual abilities* level of Harrow’s taxonomy. It also involves cognitive abilities. Students must know the properties of right, acute, and obtuse angles in order to draw them. Knowing the properties and drawing the angles fits under Bloom’s *application* level, Gagne’s *rule* level, and van Hiele’s *Level 2: Analysis.*

**Target 7:** *Students will classify triangles as equilateral, scalene, isosceles, right, acute, and obtuse.* Like Target 5, this target falls under the Dimensions of Learning Model *Reasoning Strategy 2: Classification.* It is also Bloom’s *synthesis* level, van Hiele’s *Level 3: Abstraction,* and Gagne’s *defined concepts* level.

**Target 8:** *Students will effectively communicate their mathematical reasoning orally and in writing.* This target fits in the Dimension of Learning Model under *effective communication.* Students must be able to express their ideas clearly so that teachers and other students can understand their mathematical thinking.
Instruction

Instruction for this geometry unit would include many activities from the textbook and other resources that teachers have on hand. I would expect to see fifth grade students building polygons with straws and twist ties, sorting cardboard cut-out shapes using Venn diagrams, and creating shapes on geoboards. There would be discussion focused on the attributes of the shapes. Discussion would be facilitated by the teacher so that important mathematical concepts could be learned. I am able to show glimpses of a few activities in my Captivate presentation. The quadrilateral sorting activity came from *Navigating through Geometry* (Gavin, Belkin, Spinelli, & St. Marie, 2001). There is a CD that comes with the book that includes applets. The virtual geoboard activities came from the *National Library of Virtual Manipulatives* (Utah State University, 1999).

Assessment

I created at least one assessment item for each of my learning targets. Some of the targets required more than one assessment item in order to assess the target completely.

*Completion, Short-Answer, and True-False Items*

Student performance on Target 2 required lower-order thinking skills (recall, knowledge, concrete concepts). Completion, short-answer, and true-false questions are useful for assessing lower-order thinking skills. I opted for true-false questions over short-answer or completion questions because true-false questions are easier to score. It is impossible to anticipate every response that students will make on short-answer or completion questions. Spelling errors could also get in the way when using Captivate as a scoring device. True-false questions were the most reliable in terms of scoring. They are also relatively easy to write, and they can be scored objectively by anyone with an answer key or by a machine. Using true-false questions also
allowed me to assess if the students knew the properties of the shapes (number of sides and angles) without seeing a picture of the shape. I wanted to assess the students’ knowledge of the number of sides and angles on a shape (van Hiele’s Level 2: Analysis), not if they knew what the shapes look like (van Hiele’s Level 1: Visualization). I created the following true-false questions to assess student performance on Target 2.

**Assessment 1: True or False**

1. A hexagon has 7 angles. (F)
2. An octagon has 8 sides. (T)
3. A pentagon has 5 sides. (T)
4. A quadrilateral has 6 angles. (F)
5. A triangle has 3 sides. (T)

I ordered the true-false questions alphabetically by the name of the shape. If I ordered them by the number of sides/angles as they are written in the target, it might give students a clue to the correct answer. I used the word *sides* in some of the questions and *angles* in the others.

Polygons have the same number of sides as angles. I want the students to know this and not just know the number of sides. I created both true and false statements. There is no pattern to the answers. I also made sure that no number was repeated twice because a savvy student would know that each of the polygons should have a different number of sides and angles.

True-False items can also be used to assess higher-level thinking skills. I created some true-false items to assess Target 7 which falls under van Hiele’s *abstraction* level, Bloom’s *synthesis* level, Gagne’s *defined concepts* level, and the Dimension of Learning Model’s *classification* level.
Assessment 2: True or False

1. A right triangle has one right angle. (T)
2. An obtuse triangle has two obtuse angles. (F)
3. An acute triangle has three acute angles. (T)

In order to answer these questions students must understand the definitions of right, acute, and obtuse angles and be able to think about the relationships of the three angles within a triangle. They must realize that a triangle can only have one angle that is 90 degrees or greater and still maintain the properties of a triangle. These questions cannot be answered simply by recall, unless the exact same questions have been asked and answered in class.

Multiple Choice Exercises

I created multiple choice items to assess Targets 4 and 5. All of the questions are of the multiple-response variety. I found this type of multiple-choice question helpful to assess the class inclusion aspect of the quadrilaterals. I wanted students to know multiple names for each shape, based upon the definitions of the names.

Assessment 3: Multiple Choice

1. Choose all the correct names for this shape.
   A. parallelogram
   B. rhombus
   C. square
   D. trapezoid

2. Choose all the correct names for this shape.
   A. parallelogram
   B. quadrilateral
   C. trapezoid
   D. triangle
3. Choose all the correct names for this shape.
   A. parallelogram
   B. rectangle
   C. rhombus
   D. square

The stems of all three multiple choice questions are the same. The alternatives are slightly different for each question and listed in alphabetical order. These questions are easy to score and were relatively easy to create. I rotated the square and the rectangle slightly so that students on van Hiele’s level 1 (visualization) would not be able to identify the shape. Only students on Levels 2 and 3 will be able to answer these questions correctly. If a student did not find all the names for a given shape, I would give partial credit. For example, in question 1, if a student only marked “square,” I would give him 1 point. (The entire question would be worth 3 points.) Captivate did not allow me to score this way.

These questions match instructional practices. In one fifth grade activity, the teacher holds up a shape and asks the students to think of all the names of the shape they can. When a name is given, the teacher asks why that name fits. For example, if a student said the shape was a quadrilateral and the teacher asked, “Why is it a quadrilateral?” The student would answer, “Because it has four sides.”

More assessment information could be obtained from these questions if the students were asked to explain why they chose the answers they did. This would make the items difficult to score, however. The scoring would be more subjective, and therefore, less reliable. The questions would take the students more time to complete and the teacher more time to score. Given these drawbacks, I decided to use the multiple choice questions in their current form.
Matching Exercises

I created a matching exercise to assess Target 1. The matching assessment was easy to create. If I had created multiple choice questions, I would have needed at least four questions to assess the entire target, and I would have had to repeat alternatives.

Assessment 4: Matching.

Directions: Match each shape in Column A to its name in Column B. Write the letter of the name of the shape on the blank to the left of the shape. You can use each name more than once.

___ 1.  A. circle
___ 2.  B. rectangle
___ 3.  C. square
___ 4.  D. triangle
___ 5. 
___ 6. 

Answer Key: 1C, 2B, 3D, 4A, 5D, 6B

There are more premises than responses in this matching exercise. Perfect matching reduces the validity of the assessment, because if a student knows 5 of the 6 answers, he automatically gets the last one correct.
Matching exercises are sometimes criticized because teachers only use them to assess rote memorization of terms or dates. This matching exercise was on a low level because students were just matching shape names to pictures. Target 1 was a low-level target (Bloom’s knowledge, van Hiele’s visualization, Quellmalz recall). I created a matching exercise on a slightly higher level to assess Target 7.

**Assessment 5: Matching (Masterlist items)**

Directions: Examine each triangle below. On the blank next to each triangle, write the letter:
- A if the triangle is equilateral
- B if the triangle is isosceles
- C if the triangle is scalene

Answers: 1 C, 2 B, 3 C, 4 C, 5 A, 6 C, 7 A, 8 B

This matching exercise used a masterlist format. The students need to label each triangle as equilateral, isosceles, or scalene. The exercise assesses students’ abilities to classify triangles according to specific criteria (the length of the sides), provided that the triangles pictured aren’t the same triangles that were studied in class. Students must know that the terms equilateral, scalene, and isosceles refer to the relationships among the length of the sides of the triangle. They must then analyze each triangle to determine those relationships and give the appropriate term.
I created two essay assessment tasks. These tasks help me to assess Targets 3, 7, and 8.

Assessment 6: Essay

1. Is it possible for a right triangle to have two right angles? Explain why or why not. You may include pictures in your response.
2. Discuss the similarities and differences between the two shapes below. Give at least two similarities and two differences. Use the geometry vocabulary we have learned in class to name and describe the shapes.

![Shapes](image)

Essay questions are difficult to grade. There is often low scoring reliability because the scoring is subjective. Rater drift can occur. This is when the scoring criteria change over time as the teacher grades the papers. One way to combat this is for the teacher to stop periodically and reread one of the earlier papers that was graded. How does it compare to the more recently graded essays? Another problem with scoring reliability is the halo effect. The halo effect occurs when a teacher’s general impression of a student affects how she grades the essay task. One way to combat this is to remove the names from the papers. Yet another problem is called carryover effect. This happens when there is more than one essay question to grade. A teacher’s judgment of the student’s response to the first question affects how the teacher scores the response to the second question. The way to combat this problem is to score all students’ responses to the first question, then score all the responses to the second question.

Creating scoring rubrics and training teachers to score the essays can also help to raise the scoring reliability. The two essay questions I created assess multiple targets. Since the questions are assessing mathematics knowledge and communication skills, I have created a two-
part rubric for each (see Appendices C and D). I did this by writing answers to the questions myself. Then I analyzed my answers to determine what I was looking for in student responses. Both of the rubrics are holistic scoring rubrics.

*Higher-Order Thinking, Problem Solving, and Critical Thinking*

I created a higher-order thinking task to assess Targets 4 and 5. This task assesses defined concept learning using strategy 4 which requires students to identify relationships (Nitko, 2004, p. 205).

*Assessment 7: Assessing Defined Concept Learning. Strategy 4*

Place the following words in the diagram below to show the relationships among the shapes.

Kite, Rectangle, Rhombus, Square, Trapezoid

In this task students must analyze the properties of quadrilaterals to determine the relationships and fit the terms in the graphic. This graphic representation is different from the Venn diagrams that students used in class. One disadvantage of this task is that it does not require the students to articulate the relationships among the terms. If I asked the students to
explain how they know where the terms belong, I would gain more information about their understandings. Then the assessment would become harder to grade, requiring a rubric, and it could not be assessed in Captivate.

I created the quadrilateral diagram assessment in Captivate by using a fill in the blank page. It is relatively easy to score because there is one correct answer per box with the exception of rhombus and rectangle which are interchangeable. On a written exam, I would give one point for each correct answer. Captivate does not do this. It counts the entire question as one. Another problem with Captivate is that a student could put rectangle or rhombus in both boxes and still get the answer correct.

*Performance, Portfolio, and Authentic Assessments.*

Target 6 can only be assessed using a performance assessment. In order to evaluate whether a child can draw an angle to certain specifications, you have to ask the child to draw an angle. Assessment # assesses a student’s ability to draw and measure angles.

*Assessment 8: Performance Assessment*

Using a straight edge, draw one of each type of angle: right, acute, and obtuse. Use a protractor to measure each angle. For each angle you draw: 1) Name the angle using letters. 2) Label the angle as *right, acute, or obtuse.* 3) Give the measurement of the angle.

Scoring (10 possible points)
- 1 point for accurately drawing each angle: right, acute, obtuse (3 pts possible)
- 1 point for correctly naming each angle with letters (3 pts possible)
- 1 point for accurately measuring each angle to the nearest degree (3 pts possible)
- 1 point for overall presentation, organization, and neatness of paper

This task integrates knowledge and skills. Students must know what right, acute, and obtuse angles are (Bloom’s knowledge level). They must be able to physically use a straight edge
and a pencil to draw the angles (Harrow’s perceptual abilities level). They must understand how
to label an angle and how to measure it using a protractor.

One disadvantage of using performance tasks is that they are difficult to score. The
scoring is subjective and takes a greater amount of time than fixed response items (i.e., true/false,
multiple choice, etc.). I have created an analytic scoring rubric for Assessment 8. It scores
separate parts of the task. The sum of the parts gives the total score. Creating the scoring rubric
makes the assessment less subjective and less time-consuming to score.

Target 8 also requires a performance assessment due to the nature of the target. In order
to assess students’ oral communication of their mathematical reasoning, teachers must listen to
what students say. I did not create a test item for this part of the target because I believe it must
be assessed through teacher observation over time within mathematics lessons. Interviews with
individual students or small groups of students could also be used to assess effective oral
communication skills. I used ideas from Nitko (2004, pp. 266-266) and Principles and Standards
for School Mathematics (National Council of Teachers of Mathematics, 2000) to create the
checklist in Appendix B. This checklist would help the teacher assess individual students’ oral
communication over time. I think that it would be too cumbersome for a teacher to use the
checklist for every child, but she could use it to assess specific children about whom she was
concerned. The teacher may also keep anecdotal records.

I chose not to use portfolio assessment for this geometry unit, but I would suggest that
teachers use portfolio assessment over the course of the school year to collect samples of the
students’ mathematics work. The portfolio would be a best-works portfolio. Teachers would ask
students to select a paper or project from each unit of mathematics study. The students would
select their best work. Students would write a self-assessment for each piece of work that told
why they chose that piece and what the work shows about their knowledge of mathematics. Portfolios could be shared with parents at the end of the school year to show students’ mathematical growth over time.

Validity

Validity is about the confidence we have as we interpret and use the results of a test. You can’t ask, “Is this test valid?” You have to ask, “For whom is this test valid?” Validity is a value judgment that starts and ends with the construct (the cloud). In my geometry unit, the construct would be “develop mathematics ability,” the next step in the chain would be the curriculum (geometry) and then the learning targets and the instruction. The assessment follows from the learning targets and instruction. The data that is gathered from the assessment can be analyzed and used to make decisions about selection, placement, grades, instructional strategies, etc. This chain becomes a big circle because it leads right back to the construct. Validity can be attacked at any level of the chain. I will briefly discuss some validity issues with my geometry project.

*Content Representativeness and Relevance*

This type of validity asks if the assessment got a true sample of the content that was taught. It is impossible to test everything in the curriculum, so the sampling of questions on the test needs to be fair. I asked at least one question about each learning target. The questions are aligned with the instruction. I tried to make each assessment item relevant and meaningful.

*Substantive Evidence*

This type of validity asks if the thinking skills required on the assessment are the same thinking skills and processes expected as learning outcomes. I believe my test is strong on this type of validity. The thinking skills of the assessment match the thinking skills of the targets.
Internal Structure Evidence

This type of validity is about the test. Do the questions work together? Does each task contribute to the overall result? I claim that my test assesses fifth graders knowledge of geometry. Does it really? Or do things like reading ability get in the way.? I think my test is a valid measure of fifth graders knowledge of geometry. The test could be read to them if necessary. Each question contributes to the overall effectiveness of the test. One way to test the internal validity would be to pair test items and do a correlation to see if student scores on the paired items correlated.

External Structure Evidence

This type of validity asks how my test compares to similar test. If I could pull out the geometry items on the fifth grade Virginia SOL test, I could correlate the students’ rankings on both tests. If there was a positive correlation, there would be evidence that my test had external validity.

Reliability Evidence

Reliability is part of validity, but I will also discuss it in a separate section. A test that is not reliable cannot be valid. But a test that is not valid, could be highly reliable. There are three issues with reliability: 1) would the same students receive the same scores over time; 2) if different people graded the same test, would the grades be the same; and 3) if there are two forms of the test, would the same student achieve the same result on both forms?

Generalization Evidence

This type of validity asks if students from different populations would achieve the same result on the test. I would expect all fifth graders to achieve in similar ways on my test. If they didn’t, then I might have a problem with generalization. The problem might be with the type of
instruction they received, however. If the instruction was the same and all students were taught the same learning targets in the same ways, then I would expect generalization to be high.

Consequential Evidence

This type of validity asks about the intended and unintended consequences of the test. My test is not a high stakes test, so the consequences should be minimal. I would expect a teacher to use more than one type of assessment to determine report card grades. Therefore, I would not expect this test to be used as the sole determinant of report card grades. The results of this test could be used to guide instruction and to place students in small groups within the classroom setting. I don’t anticipate any unintended consequences or negative effects from using this geometry test. Consequential validity should be high.

Practicality Evidence

This type of validity is about the cost of the test. It is measured in time and money. I designed the test with many true-false, matching, and multiple-choice questions to conserve students’ and teachers’ time. There are two essay items and one performance task that will take the students time to complete and the teachers time to grade. I believe the test is efficient and easy to use. One way to determine the practicality validity would be to ask some teachers to use the test and give feedback on it’s usefulness.

Reliability

Reliability, like validity, does not refer to the testing instrument. It refers to the results or scores on that instrument. I would expect students’ scores on my geometry test to stay consistent in a variety of situations.
Test/Retest Reliability

If I gave the test to the class and then gave it again one week later, I would expect the scores to remain consistent. Scores may even go up if the students learned something by taking the test the first time. I could find a correlation between the two sets of test scores. A correlation score (reliability coefficient) is always between 0 and 1. A score of 1 means the scores matched up perfectly. With the test/retest situation that I just describe, a reliability coefficient between 0.80 and 0.90 would show high test/retest reliability.

Alternate Forms Reliability

If I created an alternative form of my geometry test, I would expect students’ scores to be consistent if they took both tests. If I gave a different form of the test to each of two groups of students that were matched in ability, ethnicity, and socio-economic status; I would expect the scores from the two groups to correlate. There are statistical tests of correlation that I could run on either of these situations.

Inter-Rater Reliability

Inter-rater reliability is when multiple scorers score the same test and rate the test consistently. It is especially important when the test includes essay questions, open-ended questions, performance assessments, and portfolio assessments. My geometry test has essay questions and a performance assessment. If different teachers were going to be scoring the test, I would have both teachers score the same students’ test and then correlate the two scores to determine the inter-rater reliability. I would want the raters to be consistent in their rankings of students. The students may not get the same scores from both raters, but they would be ranked in the same order. One way to increase the inter-rater reliability is to train the raters so they have a consensus about the scoring. A well-written rubric is also helpful.
Ways to Improve Reliability

I could improve the reliability of my geometry test by adding more test items, taking out the open-ended questions, having two teachers score each test, and making sure that all students have enough time to complete the test. Longer assessments are more reliable than shorter ones, but creating more items for my test was not feasible given the time constraints. There are also time constraints in taking the test and grading the test which would make adding more items undesirable. Objective tests are more reliable than subjective ones. In this case, I don’t believe that removing subjective items would be worthwhile. Those items give the teacher information about the students’ mathematical knowledge that couldn’t be obtained from a closed-response test. The scoring rubrics increase the reliability. Having two teachers score the tests and averaging the scores would increase reliability but would not be efficient use of teachers’ time. A better solution is to have all the teachers using the test to score a handful of students’ responses and discuss the results. This would increase the reliability, but wouldn’t require double scoring for all of the students’ responses. I definitely would want to provide enough time for all students to finish the test. The test could be given over two class periods if necessary.

Conclusions

The process of creating my geometry assessment and writing this paper has helped me to understand the complexities of educational assessment. There are many forms of assessment and many types of decisions that can be made from assessment results. I now understand how validity and reliability issues become a balancing act involving efficiency, consistency, and values. In this era of No Child Left Behind and high-stakes testing, knowing the issues around educational assessment will help me to serve Fairfax County Public Schools as a teacher, a staff developer, and a policy maker.
References


Appendix A

Geometry Learning Targets

Educational Goal

Students will use geometry to represent and make sense of the world.

General Learning Targets

1. Students will use mathematical reasoning to analyze and justify properties and relationships among shapes.
2. Students will visualize, draw, measure, compare, and classify geometric shapes.

Specific Learning Targets

1. Students will recognize and name plane geometric figures (triangle, square, rectangle, and circle).
2. Students will identify plane geometric figures (triangle, quadrilateral, pentagon, hexagon, and octagon) according to the number of sides and angles.
3. Students will compare and contrast plane geometric shapes.
4. Students will analyze the properties of quadrilaterals (squares, rectangles, parallelograms, rhombi, kites, and trapezoids).
5. Students will classify quadrilaterals according to their properties.
6. Students will measure and draw right, acute, and obtuse angles.
7. Students will classify triangles as equilateral, scalene, isosceles, right, acute, and obtuse.
8. Students will effectively communicate their mathematical reasoning orally and in writing.
Appendix B

The van Hiele Levels of Geometric Reasoning

Adapted from *Understanding Geometry* (Richardson, 1999, p. 4) and the *Mathematics Standards of Learning Curriculum Framework* (Virginia Department of Education, 2002)

**Level 0: Pre-recognition**
Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

**Level 1: Visualization**
At this level, children see a geometric figure as a whole and would describe a shape by what it looks like. Students have no awareness of parts of figures or relationships between components of a figure.

**Level 2: Analysis**
At this level, students can describe the properties of the figure, but the properties are isolated and unrelated. For example, they might say that a rectangle has four sides and the opposite sides are parallel.

**Level 3: Informal Deduction (Abstraction)**
At this level, students are able to define a particular shape in a generalized way and can begin to tell when they have enough information to have a definition rather than just a description. They can begin to see that some shapes are a subgroup of another set of shapes.

**Level 4 Formal Deduction**
At this level, the significance of axioms, definitions, theorems, and proofs are understood. (Most high school geometry courses start at this level.)

**Level 5: Rigor**
At this level, the idea that there can be different geometries based on different axioms is understood and comparisons between the different geometries can be made.
Appendix C
Scoring Rubric for Essay Question 1
Adapted from Nitko (2004, pp. 267, 272, 282)

Score Level = 4
Mathematical Knowledge
- States that it is not possible to make a right triangle with two right angles
- Shows a complete understanding of the mathematics properties that make it impossible
- Uses appropriate mathematical vocabulary: right angle, right triangle, parallel, perpendicular, base, etc.
Mathematical Communication
- Gives a complete response with a clear explanation.
- May include an appropriate diagram
- Presents a strong argument which is logically sound

Score Level = 3
Mathematical Knowledge
- States that it is not possible to make a right triangle with two right angles
- Shows a nearly complete understanding of the mathematics properties that make it impossible
- Uses some appropriate mathematical vocabulary: right angle, right triangle, parallel, perpendicular, base, etc.
Mathematical Communication
- Gives a fairly complete response with reasonably clear explanation.
- May include an appropriate diagram
- Presents an argument which is logically sound but may contain some gaps

Score Level = 2
Mathematical Knowledge
- States that it is not possible to make a right triangle with two right angles
- Shows some understanding of the mathematics properties that make it impossible
- Uses some appropriate mathematical vocabulary: right angle, right triangle, parallel, perpendicular, base, etc.
Mathematical Communication
- The explanation may be somewhat unclear or incomplete
- May include a diagram that is unclear or incomplete
- Argument may be incomplete or logically unsound

Score Level = 1
Mathematical Knowledge
- May state that a triangle with two right angles is possible or be unsure of correct answer
- Shows limited understanding of the mathematical properties involved
- Uses limited or inappropriate mathematical vocabulary
Mathematical Communication
- The explanation may be somewhat unclear or incomplete
- May include a diagram that is flawed, unclear, or incomplete
- Argument may be incomplete or logically unsound

Score level = 0
Mathematical Knowledge
- May state that a triangle with two right angles is possible or give no answer
- Shows no understanding of the mathematical properties involved
Mathematical Communication
- Communicates ineffectively; words do not reflect the situation
- May include drawings that misrepresent the situation
- Argument may be missing or logically unsound
Appendix D

Scoring Rubric for Essay Question 2

Adapted from Nitko (2004, pp. 267, 272, 282)

Score Level = 4
Mathematical Knowledge
• Gives two similarities and two differences
• Shows a clear understanding of mathematics properties and relationships among quadrilaterals
• Uses appropriate mathematical vocabulary
Mathematical Communication
• Gives a complete response with clear descriptions
• Presents a strong argument which is logically sound

Score Level = 3
Mathematical Knowledge
• Gives two similarities and two differences
• Shows a nearly clear understanding of mathematics properties and relationships among quadrilaterals
• Uses some appropriate mathematical vocabulary
Mathematical Communication
• Gives a fairly complete response with reasonably clear explanation.
• Presents an argument which is logically sound but may contain some gaps

Score Level = 2
Mathematical Knowledge
• Gives at least one similarity and one difference
• Shows some understanding of the mathematics properties and relationships among quadrilaterals
• Uses some appropriate mathematical vocabulary
Mathematical Communication
• The explanation may be somewhat unclear or incomplete
• Argument may be incomplete or logically unsound

Score Level = 1
Mathematical Knowledge
• Gives at least one similarity or one difference
• Shows limited understanding of the mathematical properties and relationships among quadrilaterals
• Uses limited or inappropriate mathematical vocabulary
Mathematical Communication
• The explanation may be somewhat unclear or incomplete
• Argument may be incomplete or logically unsound

Score Level = 0
Mathematical Knowledge
• Gives no similarities or differences
• Shows no understanding of the mathematical properties and relationships among quadrilaterals
Mathematical Communication
• Communicates ineffectively; words do not reflect the situation
• Argument may be missing or logically unsound
Appendix E

Oral Communication of Mathematical Reasoning

Checklist

Student’s name: ________________________________________

<table>
<thead>
<tr>
<th>Behavior</th>
<th>Dates Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student contributed to whole group mathematics discussion.</td>
<td></td>
</tr>
<tr>
<td>Student contributed to small group mathematics discussion.</td>
<td></td>
</tr>
<tr>
<td>Student participated in mathematical discussion with a partner.</td>
<td></td>
</tr>
<tr>
<td>Student used appropriate mathematics vocabulary.</td>
<td></td>
</tr>
<tr>
<td>Student’s mathematical ideas were presented in an organized way.</td>
<td></td>
</tr>
<tr>
<td>Student described a solution strategy that was mathematically sound.</td>
<td></td>
</tr>
<tr>
<td>Student gave clear explanation or description.</td>
<td></td>
</tr>
<tr>
<td>Student evaluated the mathematical thinking of another.</td>
<td></td>
</tr>
<tr>
<td>Student presented strong supporting arguments.</td>
<td></td>
</tr>
<tr>
<td>Student gave example and/or counter-examples.</td>
<td></td>
</tr>
</tbody>
</table>