Precautionary Saving: A two-period illustration

Precautionary saving–the desire to build up a buffer stock of wealth when future income is uncertain–is a classic concept in dynamic economic theory. But as Blanchard and Fischer note (p. 290), if utility is constant relative risk aversion–the most common specification in quantitative macroeconomics–it's typically impossible to solve explicitly for optimal consumption. This makes it hard to get quantitative or even qualitative intuitions for a key idea in macroeconomics.

Here's an explicit solution for an exceptionally simple case, one that still illustrates the key economic ideas: It's a two period problem, with logarithmic utility, a net interest rate of zero and no discounting. To make it even more tractable–and to highlight the precautionary savings channel–income arrives only in the future: Income is a Bernoulli process with a 50-50 chance of either the high or the low outcome. This may be a reasonable approximation for the future income path of the students solving this problem: We are, I am told, living in a world where *Average is Over* and where the variance of expected income is high among the highly skilled. Here's the utility function, with all budget constraints included:

$$U(c, c') = ln(c) + 0.5ln(y_L - c) + 0.5ln(y_H - c)$$

Here y_L is the income in the low state, y_H is high state income, and c is first period consumption. This implies that expected future consumption is average lifetime income minus current consumption, or $E(c') = \frac{y_H + y_L}{2}$-$c$. One can expect that since the marginal utility of consumption becomes infinite as consumption goes to zero in either period, this agent will be obsessed with the possibility that y_L might be so low that the second period could turn out to be a consumption disaster in the spirit of Barro-Rietz.

Given these assumptions, and denoting expected income as $\bar{y}$ and the standard deviation of income $\left(\frac{y_H - y_L}{2}\right)$ as σ_y, optimal first period consumption is:

$$c^* = 0.75\bar{y} - 0.25\sqrt{\bar{y}^2 + 8\sigma_y^2},$$

The eight really stands out, doesn't it? And note that this is just with log utility, so the agent isn't all that risk-averse. As you might expect, if the variance of income is small compared to expected income, the effect of the precautionary savings motive is trivial. Try an average income of 100 and standard deviation of income of 5 or 10 to see this. But when future income is wildly uncertain consumption increases massively over the life cycle: If the standard deviation of income is half of income, expected consumption rises by about 115% over a lifetime. Precautionary savings matters enormously when people face a *de facto* positive consumption constraint each period and lifetime income is highly uncertain.

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Years ago I saw an advertisement for a economics graduate program; it said that their department taught students that there's more to economics than just calculating the third derivative. Indeed there is. But it's worth recalling that the third derivative is what drives precautionary savings: And the third derivative of the utility function is fear.

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