

# **The O-Ring Sector and the Foolproof Sector: An explanation for cross-country income differences**

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**“Why do some countries produce so much more output per worker than others?”** (Hall and Jones, QJE, 1999)

**The channel discussed here:**

*Differences in worker skill matter more for countries than for individuals*

**Evidence:** *Hanushek/Kimko AER, 2000; Jones/Schneider EI, forthcoming*

**Builds on Kremer’s “O-Ring Theory” (QJE, 1993):**

*A model of “fragile output” with strategic complementarities*

**Problem:** *Predicts same return to skill across and within countries*

**What I add:**

*A second diminishing-returns sector that uses less-skilled workers as close substitutes for skilled workers:                      The “Foolproof” Sector*

## The O-ring sector's production function

Each firm produces output this way:  
(precisely following Kremer (QJE, 1993))

$$Y_{o/\varphi} = Bk^{\alpha}q^n n$$

$Y_{o/\varphi}$  : O-ring sector output per firm.

$B$  is an exogenous productivity factor *identical across countries*.

$k$  = capital per firm

$q$  = skill level of a worker,  $0 \leq q \leq 1$

$n$  = number of workers

Output passes through  $n$  hands before becoming final output.

One worker has a small impact on output—no spillovers *assumed*.

## Efficient Output in the O-ring sector

Kremer shows that it's always privately *optimal* and socially *output-maximizing* to combine workers of identical skill within the same firm:

Example: Firm 1: [2 workers,  $q = 1$ ]. Firm 2: [2 workers,  $q = 0.5$ ]

*versus*

Firms 1 and 2: [1 worker,  $q = 1$ , 1 worker,  $q = 0.5$ ]

$$\text{Output:} \quad 1^2 + 0.5^2 > 2 * 0.5^2$$

*Nothing below changes this outcome—a key condition*

*Assume free entry of O-ring firms*

In the O-ring sector, two mediocre lawyers are no substitute for one excellent lawyer.

## Equilibrium Wages in the O-ring sector

After a surprisingly long derivation, Kremer proves:

$$w_0 = (1-\alpha)Bk^\alpha q^n$$

Or, fraction  $\alpha$  of output goes to capital owners,

$(1-\alpha)$  is divided up among the firm's  $n$  workers.

*Nothing below changes this outcome—a key condition*

*If  $q$  falls by  $\varepsilon$ , wage falls by factor of  $\approx n\varepsilon$*

***Implication: Any firm offering a pay cut of less than  $n\varepsilon$  gets all the low-skilled workers.***

## The Foolproof Sector: The Labor Force

How workers are combined:

$$\hat{L}_F = q_{uF}L_{uF} + q_{hF}L_{hF}$$

Foolproof Labor Force =

quality-weighted sum of all workers

*h = high-skilled*

*u = unskilled*

In the Foolproof sector, two mediocre lawyers can provide as much service as one excellent lawyer.

*Whenever a growth regression uses average years of schooling,  
this is the implicit model.*

## The Foolproof Sector: Output and wages

*Key assumption: Diminishing returns to labor in Foolproof sector*

$$Y_F = A(\hat{L}_F)^{1-\alpha}$$

A: Level of technology, *same across countries*

$\alpha$ : Same as in O-Ring sector, only for simplicity

For workers of a given skill level ( $s$ ), the competitive wage ( $w_{FS}$ ) will equal the marginal product of their class of labor:

$$w_{FS} = (1-\alpha)A(\hat{L}_F)^{-\alpha}q_s$$

*As quality-weighted pool of workers rises, wage falls.*

***If  $q$  falls by  $\varepsilon$ , wage falls by factor of only  $\varepsilon$***

## Equilibrium between the O-Ring and Foolproof sectors

If workers of a given skill level are working in both sectors, then they must earn the same wage: Law of one price.

$$W_{OS} = W_{FS}$$

$$(1-\alpha)Bk^{\alpha}q_s^n = (1-\alpha)A(\hat{L}_F)^{-\alpha}q_s$$

**In benchmark case, this holds for high-skilled workers.**



## General Equilibrium (1 of 2):

**O-ring pins down labor wage, Foolproof pins down labor quantity**

Stay with two-skill case: Lots of high-skill workers (h), plus a few unskilled workers (u), labor inelastic.

1. High-skilled workers work *somewhere*; consider the interesting benchmark case, where they work in both sectors:

$$L_h = L_{ho} + L_{hf}$$

2. This pins down *exact* wage in O-ring sector:

$$w_{ho} = (1-\alpha)Bk^\alpha q_h^n$$

## General Equilibrium (2 of 2)

3. Quantity of O-ring workers is pinned down by Foolproof sector:

*Too few quality-weighted workers in Foolproof:  $w_{Fh} > w_{ho}$*

*Too many quality-weighted workers in Foolproof:  $w_{Fh} < w_{ho}$*

Benchmark case: Some skilled workers in both sectors.

Number of O-ring firms =

Number of skilled O-ring workers/workers per firm

4. In benchmark, Foolproof absorbs *all* unskilled workers plus enough skilled workers to keep high-skilled wage equal across sectors.

**What this world looks like,  
Ignoring capital and the Foolproof Magnet**

*$n = 5, A = B = 1$ , ignore capital*

Country 1:  $q_h = 1, q_u = 0.9$

Country 2:  $q_h = 0.9, q_u = 0.9^2$

$$w_h^A = 1, w_u^A = 0.9$$

$$w_h^B = 0.59, w_u^B = 0.53$$

*Big returns to skill across countries.* (As seen by growth econometrician)

*Low returns to skill within countries.* (As seen by labor econometrician)

**But things aren't *that* bad in Country 2: They're worse!**

*Two forces at work in Country 2*

1. Capital multiplier in O-ring sector:

- Low-skilled countries lose lots of capital
- Low-skilled countries become less productive
- Country 2 wages fall lower than 0.59

2. Fixed Total Factor Productivity (A) in Foolproof sector:

*The Foolproof Magnet*

- Lower O-ring wages lure skilled workers  
into Foolproof sector
- Diminishing returns in Foolproof sector  
lower average productivity

*Result: Productivity plummets in Country 2 below Kremer's levels*

## Net results of Benchmark model : A review

In each country, the best workers work in O-ring sector.

*Why?*

*Because less-skilled workers would rather work in Foolproof.*

The Foolproof sector is “attached” to the O-Ring sector.

*Why? Because skilled workers can and do work in both sectors.*

In each country, unskilled workers earn a wage that is *slightly* less than that country's skilled workers.

*Why? See previous two answers.*

***Example: 30% difference in skill yields a 30X productivity gap:***

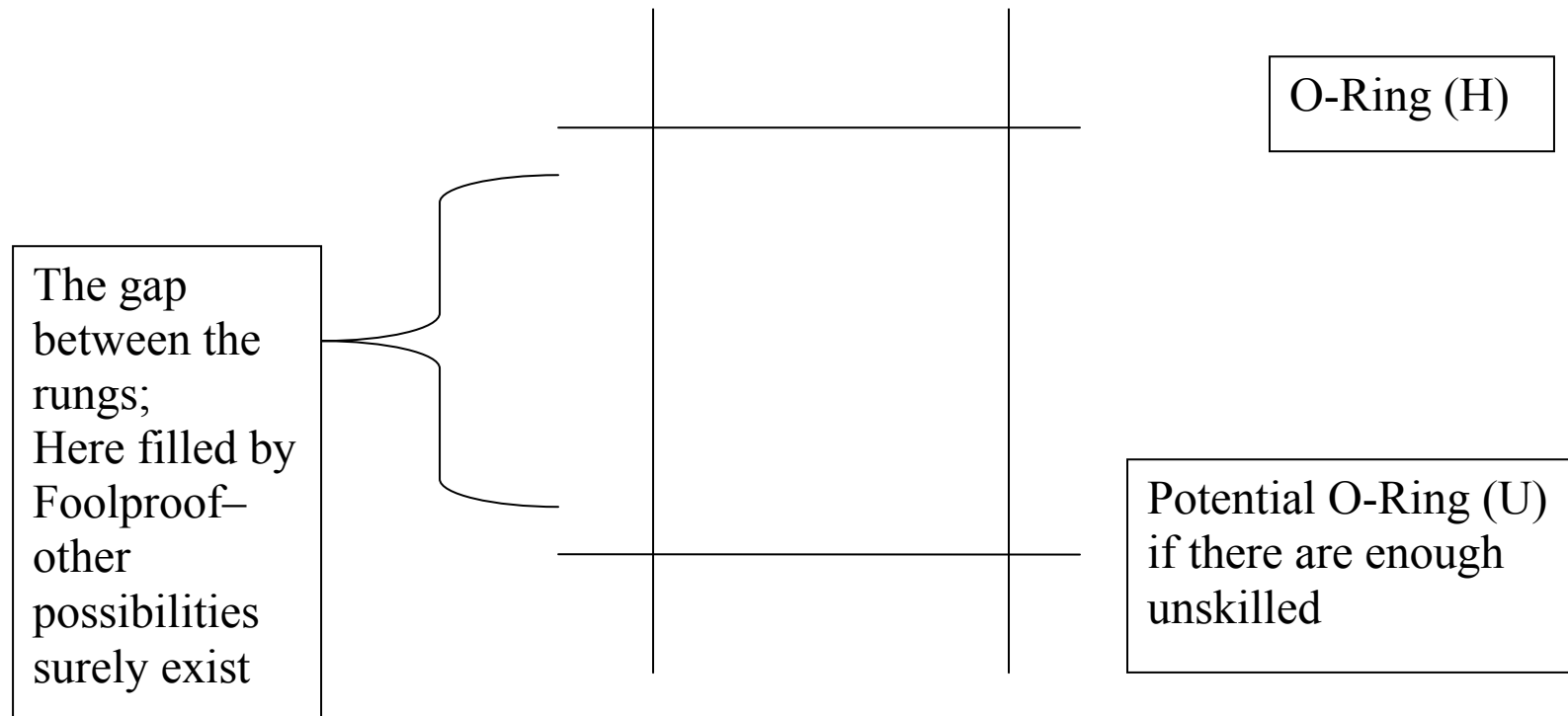
**$A=B=100$ ,  $n=3.8$ ,  $r=0.04$ ,  $L_h=1M$ ,  $L_u=100K$ ,  $q_u=0.9q_h$**

**The impact of skilled worker quality in a  
Foolproof/O-ring economy**



***This 30% skill gap would only create a 30% wage gap within a country***

## The Ladder: O-Ring Sectors as Rungs, Foolproof Sectors as gaps between rungs



## **Beyond the Benchmark:**

### **Unskilled workers out of reach of the O-Ring**

*Consider two-skill case: H and U:*

If there are too many unskilled workers,  
then Foolproof wage falls too low:

$$W_{Oh} > W_{Fh}$$
$$(1-\alpha)Bk^{\alpha}q_h^n > (1-\alpha)A(q_uL_u)^{-\alpha}q_h$$

**N.B.:** The Foolproof supply is all unskilled:

All skilled workers stay in O-ring sector.



## Out of reach of the O-Ring (2)

*What happens? Still an equilibrium: Labor markets clear.*

*If only a few too many unskilled:*

Labor econometrician sees big returns to skill:

(e.g., time-varying returns to skill in U.S.? Bigger empirical returns in LDCs?)

*If far too many unskilled:*

Wage falls to new O-Ring level:

$$w_{hU} = (1-\alpha)Bk_U^{\alpha}q_U^n$$

*Unskilled* workers in both O-ring and Foolproof sectors:

*The cycle continues*

If many workers with many levels of skill: Kremer (QJE, 1993)

*Kremer as limiting case*

## Implications

1. Low-skilled immigrants don't hurt natives.
2. Border areas as regions of Foolproofness.
3. The Flat World: Increasing the reach of Foolproofness?
4. What this tells us about education: Not an O-ring skill-builder.
5. There can be only one Foolproof Sector in each economy; and it's at the bottom.
6. These results generalize to continuous skills.
7. Empirical work can sort out "degrees of O-ringness" and "degrees of Foolproofness." (Chad Jones, working paper)
8. A naïve prediction: Life at the top should be the same everywhere.

*Barbers should earn more in the Britain than in India, but corporate executives in Britain and India should live quite similar lives.*

## Conclusion

The market abhors a vacuum:

*Diminishing (or constant) returns sectors will fill in gaps between rungs on the O-ring ladder.*

“Too easy” to match the data:

*Chad Jones (working paper, 2007) develops degrees model with degrees of O-ring-ness...*

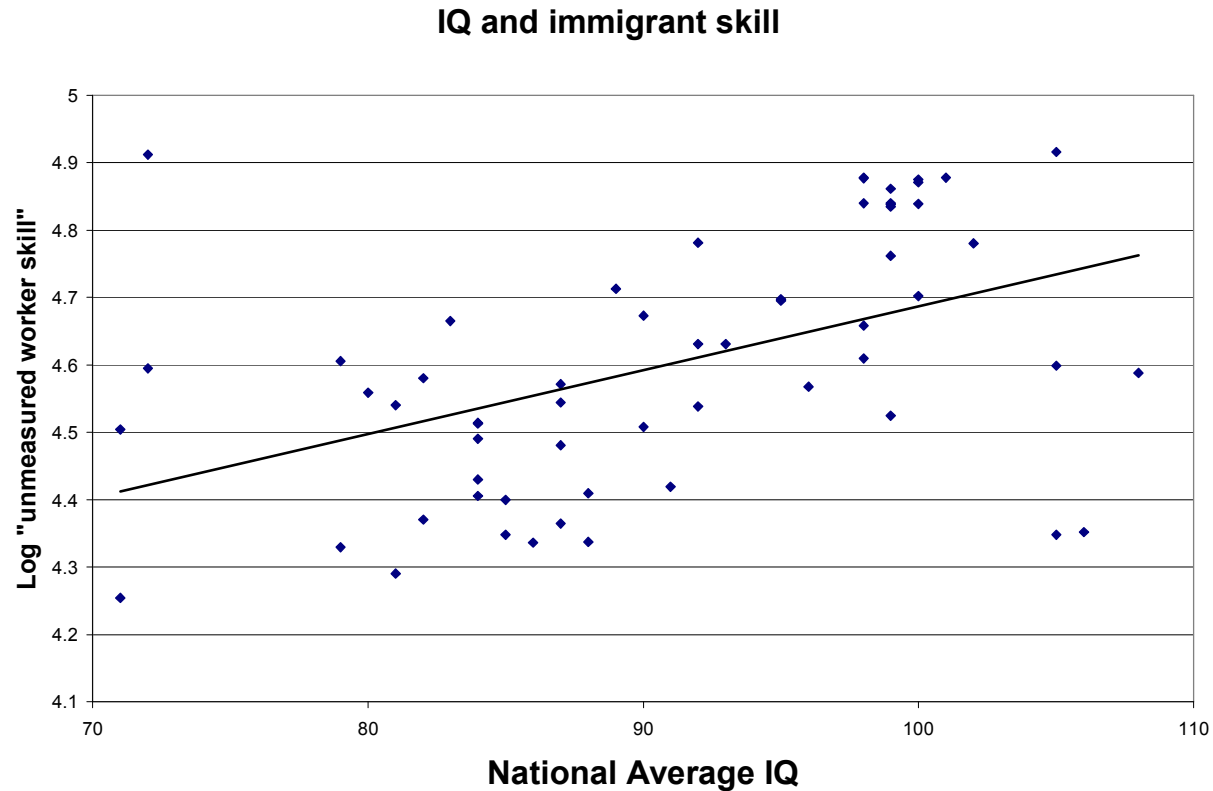
*....Can data point to the right degree?*

A story that fits some key facts...Without an appeal to externalities or variations in total factor productivity:

*Only exogenous cross-country difference: Persistent, measurable differences in individual worker skill across countries.*

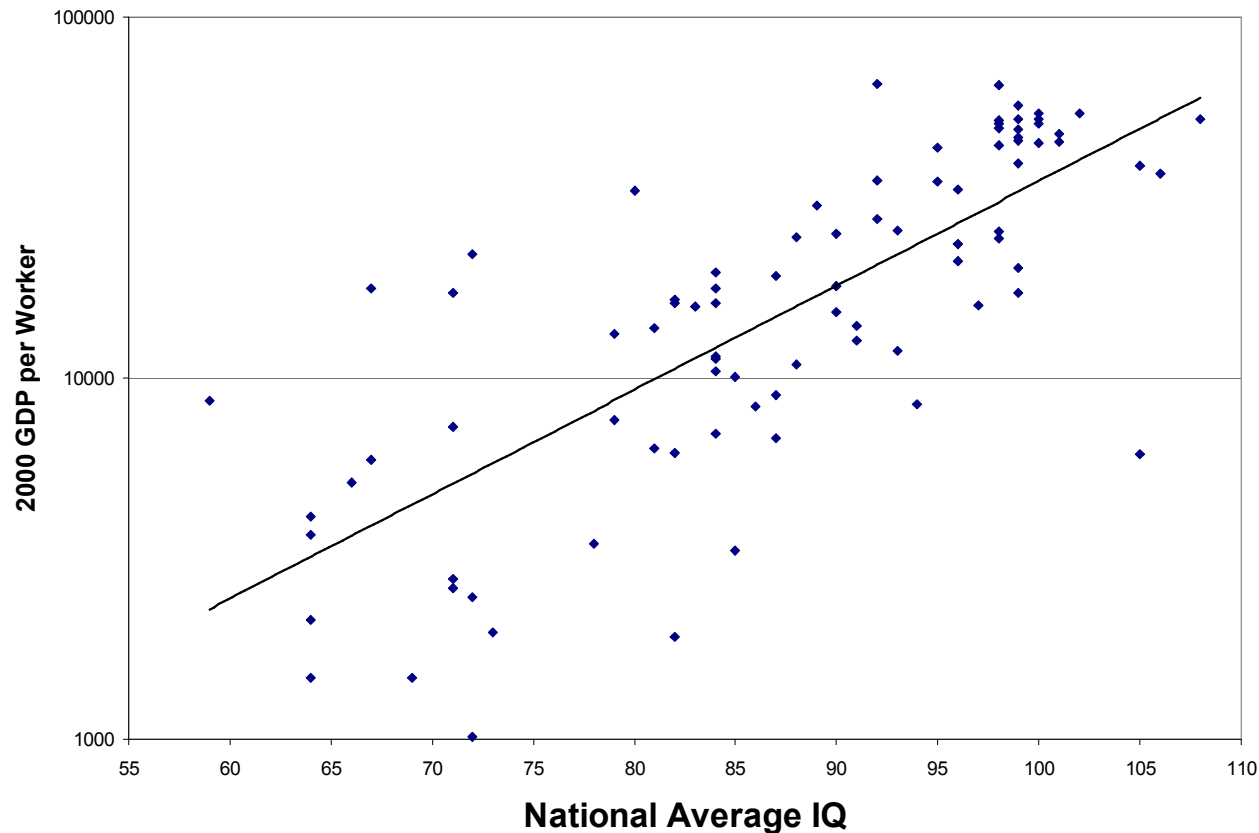
# Figure 1: IQ and Immigrant Skill

(Source: Jones and Schneider, Econ Inq., forthcoming)



Notes: The x-axis reports estimates for national average IQ for country  $i$  from Lynn and Vanhanen (2006). The y-axis reports values for  $uws_i$ , the unmeasured worker skill estimate for immigrants from country  $i$ , as estimated in Hendricks (AER, 2002).  $uws_i$  is the log average wage of immigrants for country  $i$ , adjusting for age and education. The trendline reflects the OLS coefficient of 0.95, and the  $R^2$  is 22%.

**Figure 2: National Average IQ (Lynn & Vanhanen, 2006)  
and Year 2000 GDP Per Worker**



Y-axis shows GDP per worker in logarithmic scale. Coefficient on national average IQ is 0.067, and the  $R^2$  is 58%. The outlier in the lower-right corner is China (IQ = 105).

## A detour: Capital in partial equilibrium

Kremer shows that in O-ring sector,

$$k^* = \left( \frac{\alpha q^n n B}{r} \right)^{\frac{1}{1-\alpha}}$$

Yields another a multiplier effect of quality in O-ring sector:  
Best workers get more machines, as in Cobb-Douglas world

Take  $r$  as given for simplicity. Justifications:

- Steady-state of Solow or Ramsey model
- A open-economy world with free flow of capital.