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Abstract
As a rule, rational workers prefer to tax workers, not capitalists.

Taxes Rational Workers Want: An Illustration of the Chamley-Judd Theorem

Let's assume there's a fixed amount of government, *g*, to pay for each period, and budgets are always balanced. There's a bill to pay and it can't be avoided. The question before us is simple. In the long run, in the steady state, which policy produces the highest after-tax wages for workers: an income tax on capital or a income tax on labor? Common sense says that workers would naturally prefer capitalists to pick up the tab but common sense is a bad guide to astronomy so perhaps it's a bad guide to tax policy as well.

Output is Cobb-Douglas:

$$Y = K^{0.5}$$

and labor is supplied inelastically. Markets are competitive, and since the exponent on capital is 1/2, workers and capitalists each get half of pre-tax GDP in equilibrium. Capitalists always discount the future at rate *r* per period. First consider what happens if capitalists pay the bill, and the tax rate on capital is t_k per period. The tax rate has to adjust to pay for the fixed *g*, and since capital earns half of GDP then the following must hold:

$$g = t_k 0.5K^{0.5} \qquad (1)$$

In a Ramsey-style steady state without long run technological progress, the after-tax return on capital must equal the rate of time preference, which in turn equals $1\text{-}t_k$ times the marginal product of capital:

$$r=(1\text{-}t_k)0.5K^{-0.5} \qquad (2)$$

One could well think of *r* as the after tax return available elsewhere in the global economy---so everything in this short note works equally well in an open- or closed-economy setting.

These two equations are all we need to solve for K^{*k}, the steady state capital stock under a capital tax: After substituting out t_k and a few steps of algebra (including a rare use of the quadratic formula) we find:

$$K^{*k} = \quad [(0.25 - 2gr)+(0.25^2 - (2gr)^2)^{0.5}]/(2r^2)$$

Once we know K^{*k} we know total after tax wages for workers:

$$w^{*k} = 0.5(K^{*k})^{0.5} \qquad (3)$$

The steady-state wage under a labor tax, w^{*n}, is simpler to calculate since labor is supplied inelastically and capital is now free of tax distortions. Solving (2) without a capital tax we see that the untaxed capital stock heads toward $(2r)^{-2}$ so total after-tax wages equal

$$w^{*n} = (4r)^{-1} - g. \qquad (4)$$

One quick check that we are on the right path: You will see that total wages are the same in (3) and (4) if government spending falls to zero.

Now we would like to know: Can we be sure that w^{*n} is greater than w^{*k}? You are welcome to prove that formally, but I decided to go with an Excel file. Try plugging in different values of *g* and *r* and you will find that after-tax wages are always higher when government puts the taxes on workers rather than the capitalists.

Rational workers would rather pay the tax themselves using a big capital stock rather than have the capitalists pay the tax but be forced to use a smaller capital stock.

Coda

Yes, there are plenty of ways to change this result. Examples: If capital has positive externalities or if workers are more farsighted than capitalists then rational workers might want to *subsidize* capital rather than leaving it untaxed. And Emmanuel Saez has summarized some critiques of the Chamley-Judd result that push in the direction of capital taxation. But we should start with the Chamley-Judd zero capital tax result because even though it's built on standard Ramsey model foundations it provides a shocking result that reminds us of an easily neglected fact: Capital helps workers.

Paul Samuelson said that comparative advantage is one idea in economics that is true but not obvious. The Chamley-Judd zero capital taxation result is another. Both are important starting points for serious thinking about economic policy.

Another point: *g* could just be a lump-sum grant to workers. Rational workers would turn that down rather than tax capitalists. Chamley-Judd offer a Redistribution Impossibility Theorem.

> The major finding is that if the economy were to converge to a steady state where all agents have a common rate of time preference, no agent will asymptotically choose redistributive capital income taxation, independent of his initial and asymptotic level of wealth. --Kenneth Judd, 1985

> The capital income tax is also often regarded as a useful instrument for the redistribution of income. However, the analysis in this paper, applied to an economy of heterogeneous agents, shows that when individuals have long lives, a permanent tax on capital income is not an efficient policy for redistribution.
> --Christophe Chamley, 1986

And don't forget a bonus implication: *Everything* you delay gratification for is capital, so….