Simple Examples of Buchanan (1965) Club Goods

1. Pure fixed cost + constant marginal congestion effects

A person has income Y to spend on two items: private consumer goods x and a club good. A club is a nonprofit funded by the fees of its n members which offers benefits exclusively (and excludably) to its n members. The club good becomes less appealing when it is busier (as in Buchanan's neighborhood swimming pool example). The club has to cover its fixed costs of building a pool (F) with membership fees t per person, but following Dennis Mueller's treatment in *Public Choice*, let's just substitute out the constraint that $tn=F$. Each person's utility function then looks like this in a simple setup, one that assumes in advance that the person is going to join the club:

$$\mathcal{L}(x, n) = x - \alpha n + \lambda(Y - x - F/n)$$

The constraint reflects the fact that all income will be spent on either the private good x or on membership fees $t=F/n$. The economic question of interest is what does n, optimal club size, depend upon? It might be worth checking the conditions for an interior solution, but that's likely less interesting than what drives optimal club size.

2. Constant marginal cost + diminishing marginal benefits

Let's consider a case that Buchanan explicitly said he wasn't trying to explain: The benefits of camaraderie within a club. We'll assume that each extra comrade offers fewer benefits than the previous comrade. This setup could be a shorthand for some models of network effects as formulated by Matthew Jackson or some models of social media platforms. Now fixed costs are negligible, but each member of the club imposes a marginal monetary cost γ on the rest of the club— say the cost of electricity, cleaning, refreshments, keeping the network servers running efficiently. Each person's utility function then looks like this in a simple setup:

$$\mathcal{L}(x, n) = x + \alpha n^\beta + \lambda(Y - x - \gamma n), \, 0 < \beta < 1.$$

Now what's the optimal club size as a function of the deep parameters? Bonus: Why would this problem have had a knife-edge solution if the marginal benefits of comraderie were constant, for instance αn?