Preparation for the Final Exam

Math 290 **Dr. Colonna** **June 13, 2016**

(1) State in words the contrapositive and the converse (specify) of the following statement: *If the test is easy and I study hard, then I get an A.*

(2) Write the truth table for each of the following statements and decide whether it is a tautology, a contradiction, or neither.

(a) $\sim(\sim P \Rightarrow (P\vee \sim Q))$,

(b) $(P\vee \sim Q) \Leftrightarrow \sim((\sim P)\wedge Q)$.

(3) Is the statement $\sim[(\exists x)(x > 0 \wedge x^3 = -x)]$ true in the universe of all real numbers? Explain.

(4) Let $\mathbb{R}$ be the universe. Give a useful denial of the statement $(\exists x)(\forall y)(x^2 > 0 \wedge xy \neq 1)$. Make sure not to have $\sim$ in your final answer.

(5) Provide either a proof or a counterexample for the following statement: *If a, b, c are any integers such that a divides both $2b - c$ and $b - c$, then a must divide both b and c.*

(6) Let $A = \{\emptyset, a, \{a, \emptyset\}\}$. List all elements of the power set of A.

(7) Draw a digraph for the relation R on the set $\{a, b, c, d, e\}$ given by

$$R = \{(a, a), (b, c), (c, e), (b, e), (b, a), (d, d)\},$$

and determine whether R is reflexive, symmetric, transitive, antisymmetric.

(8) Specify whether each of the following statements is true or false. Explain.

(a) $\{e\} \subseteq \{e, \{e^2\}\}$; (b) $\emptyset \in \{e, \{\emptyset\}\}$; (b) $\{\emptyset\} \in \mathcal{P}(\{\emptyset\})$.

(9) Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2\}$, $B = \{2, 3, 4, 6\}$, and $C = \{3, 4, 7\}$. List the elements of the following sets:

(a) $(A \cap B) \cup C$; (b) $A^c - (B \cap C)$.

(10) For each natural number n, let $A_n = \left[-\frac{2}{n+1}, \frac{4n-1}{3n}\right]$. Find $\bigcup_{n\in\mathbb{N}} A_n$ and $\bigcap_{n\in\mathbb{N}} A_n$.

(11) Let $a_1 = 1$, $a_2 = 7$, $a_3 = 7$, and for each natural number $n \geq 3$, let

$$a_{n+1} = 2a_n + a_{n-1} - 2a_{n-2}.$$

Use the PCI to prove that for each $n \in \mathbb{N}$,

$$a_n = 1 + 2(-1)^n + 2^n.$$

(12) Specify whether each of the following statements is true or false. Justify your answers.

(a) If the sets A, B, $A \cap B$ have 12, 7, and 4 elements, respectively, then the set $(A - B) \times (A \cup B)$ has 120 elements.

(b) A denumerable set always has a proper denumerable subset.

(c) If A is any set, then $(A \cup A) \times A = A \times (A \cup A)$.

(d) An equivalence class defined by an equivalence relation on a set X is an element of the power set of X.

(13) Let R and S be the relations on $\{a, b, c, d\}$ defined by $R = \{(a, b), (c, b), (d, c), (d, b)\}$, and $S = \{(b, a), (a, b), (c, d)\}$. Find $R \circ S$, and $S^{-1} \circ R$.

(14) Let $\mathcal{A}$ be the partition on the set $\{a, b, c, d, e, f, g\}$ defined by

$$\mathcal{A} = \{\{a, e\}, \{b, c, d\}, \{f, g\}\}.$$

List the ordered pairs in the equivalence relation associated with $\mathcal{A}$.

(15) Draw the Hasse diagram for the poset $X = \{1, 2, 5, 7, 10, 14, 35, 70\}$ under the partial ordering R defined by mRn iff m divides n.

(16) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 1 + 2x - x^2$. Find the sets $f([1, 2])$ and $f^{-1}(f([0, 1]))$.

(17) Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be given by $f(m, n) = m(2n + 1)$. Find the sets

(a) $f^{-1}(A)$, where $A = \{2, 4, 6, 8, 10\}$.

(b) $f(B)$, where $B = \{1, 2\} \times \{1, 2\}$.

(18) Prove that if $f : A \to B$ and $g : B \to C$ are both 1-1, then $g \circ f : A \to C$ is 1-1.

(19) Prove that $X = \{n \in \mathbb{Z} : n \text{ is odd}\}$ is denumerable by constructing a one–to–one correspondence between X and $\mathbb{N}$.

(20) (a) Make a table according to the categories, "finite", "denumerable", and "uncountable" and insert in the appropriate column each of the sets below.

(b) Which of the following sets are equivalent? List all possible pairs of such sets.

$$\mathbb{R}_+,\ \mathbb{Z}_-,\ (0, \sqrt{2}),\ \{3^q : q \in \mathbb{Q}\},\ \mathcal{P}((-\infty, 1]),\ \mathcal{P}(\{0, \sqrt{2}\}),\ \mathcal{P}(\mathcal{P}(\emptyset)), \mathcal{P}(\mathbb{N}), \mathbb{Z} \times \mathbb{N}.$$

(c) Order the cardinal numbers of all of the above sets from smallest to largest.