PRACTICE TEST 3

Math 290 **Dr. Colonna** **Summer 2016**

(1) Let $X = \{1, 2, 3, 4\}$ and consider the relation on X defined by

$$R = \{(2, 3), (4, 1), (3, 3), (3, 1), (1, 4)\}.$$

(a) Find the domain and the range of R.

(b) What is the minimum number of elements that need to be added to R to make it an equivalence relation on X? Specify which elements must be added and why.

(c) Let S be the equivalence relation obtained in part (b). List the elements of the equivalence class of 3.

(2) Specify whether each of the following statements is true or false. If the statement is false, give a counterexample.

(a) If R and S are relations on a set X, then the range of R is equal to the range of $S \circ R$.

(b) If R is a symmetric relation on a set X, then R cannot be antisymmetric.

(c) If A and B are sets, then $A \times B \neq B \times A$ unless $A = B$.

(d) If R is a transitive relation on X and $A \subseteq X$, then the relation S on A defined by $S = \{(x, y) \in R : x \in A \wedge y \in A\}$ is transitive.

(3) Let S be the relation on $\{a, b, c, d\}$ defined by $S = \{(b, a), (a, c), (b, b), (c, d)\}$. Find S^{-1} and the range of S^{-1}.

(4) Consider the relation R on the set $\{a, b, c, d\}$ given by

$$R = \{(a, a), (b, b), (b, c), (c, a), (b, a), (c, c)\}.$$

(a) Draw the digraph of R.

(b) Is R (1) symmetric? (2) reflexive? (3) transitive? (4) antisymmetric? Explain.

(5) Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and consider the collection of subsets of X given by

$$\mathcal{P} = \{\{1\}, \{2, 5\}, \{3, 4, 6\}, \{7\}\}.$$

List all the elements of the equivalence relation R having $\mathcal{P}$ as the set of equivalence classes.

(6) Let X be the subset of the natural numbers $\mathbb{N}$ consisting of the divisors of 36. The relation R on X defined by aRb iff a divides b is a partial ordering on X. Draw the Hasse diagram of R.

(7) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 1 - x^2$, $g(y) = \sqrt{2 - y}$. Find $(f \circ g)(y)$ and specify the domain and range of $f \circ g$.

(8) Give an example of three sets A, B, C and two functions $f : A \to B$ and $g : B \to C$ such that $g \circ f$ is 1-1, but g is not 1-1.

(9) Define $f : \mathbb{Z}_6 \to \mathbb{Z}_6$ by $f(\overline{m}) = 3\overline{m}$, where $\overline{m}$ denotes the equivalence class of m.

(a) Verify that f is a function.

(b) Prove or disprove: f is 1-1.

(c) Prove or disprove: f is onto.