Practice Test 1

Math 290 **Dr. Colonna** **May 19, 2016**

(1) Give the precise definitions of the following terms:

(a) Consequent (b) Contradiction

(c) Conditional statement (d) The converse of *if P, then Q*.

(2) Write the truth table for each of the following statements and decide whether it is a tautology, a contradiction, or neither.

(a) $\sim (P \wedge \sim Q) \Rightarrow (P \vee \sim Q)$

(b) $P \iff (\sim Q \wedge ((\sim P) \Rightarrow Q))$

(3) Write the following statements in symbolic form and give a useful denial.

(a) *For every rational number x, $2x - \pi \neq 0$ or $x < 1$.*

(b) *n is the product of two primes and its square is not divisible by 2.*

(b) *$x^2 - \pi < 0$, for every rational number x.*

(4) Which of the following are true in the universe of all whole numbers, $U = \{0, 1, 2, 3, \dots\}$? Explain.

(a) $(\exists! x)(x \geq 0 \wedge \sqrt{x} = x)$

(b) $(\forall x)(x^2 - x \text{ is even})$

(5) Which of the following are true in the universe of all real numbers? Explain.

(a) $(\exists x)(x > 0 \wedge \sin x = x)$;

(b) $(\forall x)(\exists y)(x + y < 2 \wedge x - y > 1)$.

(6) Prove that for all integers n, $3n + 1$ is odd if and only if n is even.

(7) Recall that a real number x is a rational number if and only if it can be expressed as $x = \frac{m}{n}$, where m, n are integers and $n \neq 0$. A real number which is not rational is called *irrational*.

(a) Give a direct proof of the following statement: *The product of two rational numbers is a rational number.*

(b) Let x be a nonzero rational number. Give a proof by contraposition, using part (a), of the following statement: *If y is an irrational number, then $\dfrac{y}{x}$ is an irrational number.*

(c) Let x be a rational number. Provide either a proof or a counterexample for the following statement: *For all irrational numbers y, $x - y^3$ is an irrational number.*

(8) Prove that for all natural numbers a and b, $\gcd(a, b) = a$ if and only if $a \big| b$ (that is, a divides b).

(9) Prove that if a, b, c, d are any integers such that a divides $b + c$ and a divides $c + d$, then a divides $b - d$.