# Practice Test 2

**Math 290** **Dr. Colonna** **May 29, 2019**

(1) Specify whether each of the following statements is true or false. Justify your answers.

(a) $u \in \{\{u\}, u\}$; (b) $\{u\} \subseteq \{u, \{u\}\}$;

(c) $\{\emptyset\} \subset \{\{u\}\}$; (d) $\{u\} \in \mathcal{P}(\{u\})$.

(2) Specify whether each of the following statements is true or false.

(a) $\emptyset \in \{\{\emptyset\}\}$; (b) $\{\alpha\} \in \{\emptyset, \{\alpha\}\}$;

(c) $\emptyset \subseteq \{\{\alpha\}\}$; (d) $\alpha \subseteq \{\alpha, \{\emptyset, \alpha\}\}$.

(3) Let $S = \{1, b, \{1, b\}\}$. List all the elements of the power set of $S$.

(4) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{4, 5, 6\}$, and $C = \{3, 5, 7\}$. List the elements of the following sets:

(a) $(A \cap B) \cup C$; (b) $A - (B^c \cup C)$; (c) $B \times (B \cap C)$;

(d) $A^c - (B \cap C)$; (e) $A \cup (\emptyset^c \cap C)$ (f) $(A - (B - C))$.

(5) For $n \in \mathbb{N}$, let $A_n = \left(1 - \frac{2}{n}, \frac{2n+1}{n}\right)$. Find $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$.

(6) Prove or give a counterexample for the each of the following statements.

(a) For any three sets $A, B, C$,

$$A \cup (B \cap C) = (A \cup B) \cap C.$$

(b) Let $\{B_n\}_{n \in \mathbb{N}}$ be any sequence of sets and $A$ a set. Then

$$A \subseteq \bigcup_{n \in \mathbb{N}} B_n \Longrightarrow (\exists n \in \mathbb{N})(A \subseteq B_n).$$

(7) Prove that for each natural number $n$,

$$3^n > n^2.$$

(8) Let $X$ be a set of 3 elements. Determine the number of elements of the power set of the power set of $X$ (that is, of the set $\mathcal{P}(\mathcal{P}(X))$.

(9) For each natural number $n$, let $A_n = \left[1 + \frac{2}{n}, \frac{6n-3}{n}\right]$. Find $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$.

(10) Let $a_1 = 0$, $a_2 = 6$, and for $n \geq 2$, let $a_{n+1} = 4a_n - 3a_{n-1}$. Prove that $a_n = 3^n - 3$ for all $n \in \mathbb{N}$.

(11) Prove or give a counterexample for the following statement. *If $A$ and $B$ are sets, then $A \cup B \subseteq A \cap B$ if and only if $A = B$.*

(12) Determine the coefficient of $u^3 v^4$ in the binomial expansion of $(u - 2v)^7$.