

# Introducing Preference Heterogeneity into a Monocentric Urban Model: an Agent-Based Land Market Model

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**Abstract.** This paper presents an agent-based urban land market model. We first replace the centralized price determination mechanism of the monocentric urban market model with a series of bilateral trades distributed in space and time. We then run the model for agents with heterogeneous preferences for location. Model output is analyzed using a series of macro-scale economic and landscape pattern measures, including land rent gradients estimated using simple regression. We demonstrate that heterogeneity in preference for proximity alone is sufficient to generate urban expansion and that information on agent heterogeneity is needed to fully explain land rent variation over space. Our agent-based land market model serves as computational laboratory that may improve our understanding of the processes generating patterns observed in real-world data.

## 1 Introduction

Spatial forms of cities and urban land prices are the results of land allocation between competitive users via land markets. Land market models in urban economics, as many other economic models, often assume a single representative agent [1, 2]. This paper presents an agent-based model of an urban land market in which agents exhibit heterogeneous preferences for proximity to the urban centre. We compare macro scale economic and spatial measures arising from both homogeneous and heterogeneous agents interacting in a land market. We show that by providing the opportunity to track characteristics of agents, the spatial goods being exchanged, and associated land transaction data, our agent-based land market serves as a computational laboratory for exploring micro-macro linkages in urban land markets (particularly, links between individual preferences, emerging land prices and urban patterns).

We underline the importance of building from existing theoretical and empirical work done in spatial, urban and environmental economics in constructing our ABM of land use with an endogenous market mechanism. Many traditional models of urban

land markets find their roots in the monocentric urban model of W. Alonso [1]. According to his bid-rent theory, households choose locations at a certain distance from the central business district (CBD) by maximizing utility from the joint consumption of a spatial good (a land lot or house) and a composite good (all other goods) under their budget constraint. The outcome of the bid-rent model is a set of rent gradients (i.e., land prices at different distances from the CBD). The model predicts that the land rent gradient is decreasing with distance and land prices for equidistant locations are the same.

As is typical in economics, certain restrictive assumptions are made to solve for equilibrium conditions in traditional urban economics models. In general these restrictive economic assumptions can contradict real world phenomena and have raised substantial criticisms. These assumptions (each of which has a representative example in urban economics) fall into four general areas [3]: limitations of *the representative agent* approach [4]; limitations of assumptions of economic *rationality* [5]; absence of direct *interactions among agents* [6]; and absence of analysis of *out-of-equilibrium dynamics* [5, 7, 8]<sup>1</sup>. As discussed by many agent-based computational economics scholars, ABM may serve as a tool to relax one or several of these assumptions to shift to more realistic models. For the purposes of this paper, we introduce heterogeneous agents and replace an equilibrium centralized price determination by distributed bilateral trading.

Applications of ABMs to land use (ABM/LUCC) are quite diverse [9]. To date, the majority of efforts of the ABM community to integrate markets into ABM/LUCC have been focused on agricultural applications [10-12], which differ from urban land markets [13]. Several models study the effects of hypothetical urban land markets, but with primary emphasis on the demand side. The SOME and SLUCE models allow agents to choose the parcel that maximizes their utility without competition from other sellers and assuming that the locating agent will outbid the current use [14]. The MADCM model provides a welfare analysis of the simulated urban land market [15] with the focus on the demand side. Our model moves beyond previous work in several aspects. Both the demand and supply sides are represented in detail, facilitating model experiments focused on the drivers of each<sup>2</sup>. The process of locating trading partners in space, forming bid and ask prices, and resolving trades is also modelled explicitly. The primary aim of this paper is to investigate *how aggregated land patterns and rent gradients change in the monocentric urban model if agents with homogeneous preferences for proximity are replaced by heterogeneous ones*. In the coming sections we describe the structure of our model and discuss the simulation results.

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<sup>1</sup> See [3, 13] for a wider discussion of this criticisms with respect to urban economics

<sup>2</sup> In this particular paper we replicate the monocentric urban model that assumes that sellers are agricultural land owners and that their ask price is the same for every cell. However, the code of our program integrates the possibility to model the formation of ask prices for households and agricultural sellers.

## 2 The models

Our *Agent-based Land Market (ALMA)* model explicitly represents micro-scale interactions between buyers and sellers of spatial goods. In line with the assumptions of the monocentric model, the ALMA model assumes that sellers (i.e. owners of agricultural land) offer land at the same fixed price equal to agricultural opportunity costs and that each spatial good is differentiated by distance from the CBD (or its inverse measure – proximity  $P^3$ ), while environmental amenities ( $A$ ) are assumed to be distributed uniformly in the city. Buyers (i.e., households) search for a location that maximizes their utility  $U = A^\alpha \cdot P^\beta$  ( $\alpha$  and  $\beta$  are individual preferences for green amenities and proximity correspondingly and utility, as usual in micro economics, is a mathematical representation of preferences) and is affordable under their disposable budget for housing net of transport costs ( $Y$ ). The rationality of agents is bounded by the fact that they do not search for the maximum throughout the whole landscape but rather search for the local maximum among  $N$  randomly chosen cells. We impose this assumption since the search for a house in reality is very costly (time-wise and money-wise), meaning that a global optimum is not likely to be located in real-world housing markets. After defining the spatial good that gives maximum utility, a buyer forms the bid price. A bid price is a function of utility ( $U$ ), individual income ( $Y$ ) and prices of all other goods (the influence of which is expressed by a constant  $b$ )<sup>4</sup>:

$$P_{bid} = \frac{Y \cdot U^n}{b^n + U^n} \quad (1)$$

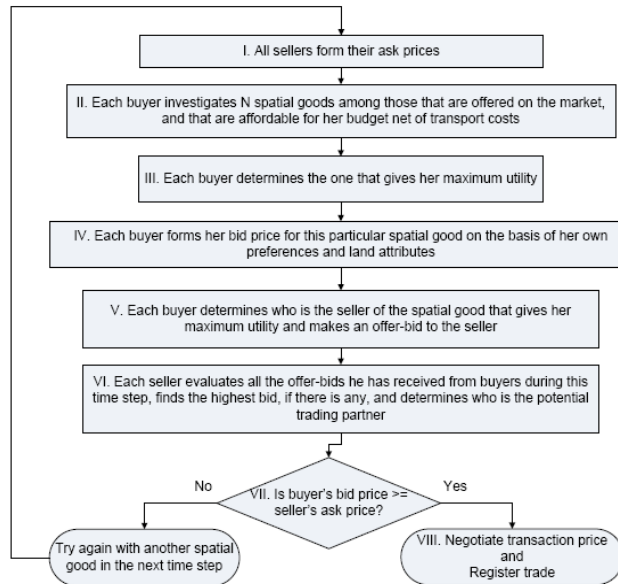
Then buyers submit their offer-bids to the sellers. Sellers choose the highest bid-offer and if it is above their ask price, then transactions take place. If not then both buyer and seller participate in the land market in the next time step. The final transaction price is an arithmetic average of the ask price and the highest bid price. Figure 1 shows the logic of the trading mechanism, i.e. one time step in the model<sup>5</sup>. The model stops running when no more transactions occur, i.e. all the submitted bids are lower than ask prices.

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<sup>3</sup> Proximity is defined as  $P = D_{max} + I - D$ , where  $D$  is distance of a cell to the CBD

<sup>4</sup> The justification and properties of this demand function are discussed in details in [3]

<sup>5</sup> For extended description of the event sequencing see [3]



**Fig. 1.** Conceptual algorithm of trade

Of course buyers and sellers are not the only agents participating in a land market. Urban developers and real estate agents influence both spatial patterns and land price formation. We discuss their roles and ways to integrate them in the ALMA model elsewhere [3, 16]. In this paper we keep the model as simple as possible to give an analysis of the effects of *agents' preferences heterogeneity* on the urban pattern and land prices.

### 3 Simulation Experiments

The model simulations produce spatially explicit rent gradients and land patterns. Experiments varying different parameters such as transport costs, bidding strategy, the level of environmental amenities and others have been performed with the model [3, 13]. In this paper we investigate how changes in buyers' and sellers' preferences, particularly a shift from homogeneous to heterogeneous preferences for proximity to the urban centre, affect economic indicators and the spatial morphology of the city. In addition to graphical representations, we also present a set of metrics to analyze micro and macro economic and spatial outcomes, including welfare measures, economic and spatial indicators, and estimated land rent gradients<sup>6</sup>.

All the model experiments presented in this paper were performed on a 29x29 grid of cells. The total number of sellers was set equal to 841 and the number of buyers

<sup>6</sup>An equation that quantitatively characterizes the transaction price at a given distance from the city centre, estimated using linear regression analysis. The land gradient is a typical characteristic of urban spatial structure analyzed both theoretically and empirically [2]

was equal to 1000. The ALMA parameters for all model experiments are listed in Table 1; the only parameter that was varied between the 2 experiments is the agent's preference for proximity. The comparison of the outcomes in terms of macro and micro economic and spatial measures is presented in Tables 3 and 4.

**Table 1.** Values of parameters in the simulation experiments

Symbol	Meaning	Exp 1	Exp 2
Y	Individual budget	800	800
A	Level of green amenities	1	1
b	Constant in (1)	70	70
$N_{\text{cells}}$	Number of spatial goods (lots) in the city	841	841
$P_{\text{ask}}$	Ask price of a seller of agricultural land	250	250
TCU	Transport costs per unit of distance	1	1
$\beta$	Individual preference for the proximity to the CBD	0.85	uniform distribution [0.7;1]
---	Mean preference in the traders population	0.85	0.85

Multiple experiments runs with different random seeds were performed. A different random seed affects both the distribution of preferences and the order of activation of agents. Difference in the morphology of a city and economic indices are small. Future work will derive formal bounds for these realized output metrics. Below we present results and provide a discussion of two typical simulations different in the settings as described in Table 1.

**Table 2.** Economic and spatial metric outcomes of the ALMA experiments

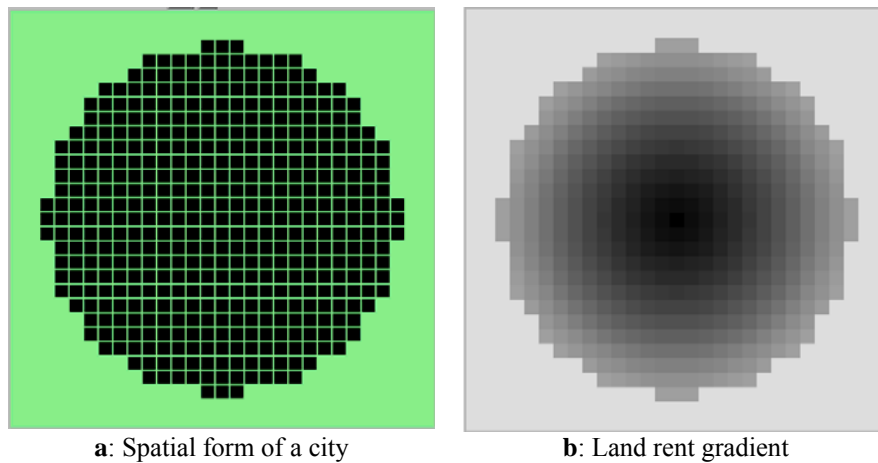
Parameter		Exp 1	Exp 2
Individual utility:	Mean	65.48	65.71
	St. dev.	12.58	13.22
Aggregate utility		30 448.82	33 312.31
Buyers' bid price:	Mean	363.72	364.28
	St. dev.	74	77.54
Urban transaction price:	Mean	306.86	307.14
	St. dev.	37	38.77
Average surplus:	Buyers'	56.86	57.14
	Sellers'	56.86	57.14
Total property value		142 690.16	155 721.10
City size (urban population)		465	507
Distance at which city border stops		12.08	12.65

**Table 3.** Linear regression estimates of rent gradient functions (Dependent variable is the transaction price)

Parameter	Exp 1	Exp 2 (1 and 2)	
		1 independent variable (D)	2 independent variables (D and $\beta$ )
Number of observations	465	507	507
R:	<i>0.9905</i>	<i>0.9560</i>	<i>0.9832</i>
Intercept:	estimate	410.76	482.32
	St error	0.501	2.477
	t-Value	819.68	194.71
Distance to CBD:	estimate	-12.81	-11.98
	St error	0.058	0.078
	t-Value	-219.94	-153.32
Buyers' preference for proximity:	estimate	--	-93.50
	St error	--	3.271
	t-Value	--	-28.58

### 3.1 Experiment 1

We begin with an experiment that replicates the benchmark case of a monocentric Alonso model with homogeneous agents (also presented in [3]). The main difference between the simulation experiment and the analytical model is that the centralized land price determination mechanism is replaced by a series of spatially distributed bilateral trades. The results are presented in Table 2. The spatial form of the city and urban land rent gradient are presented in Figures 2a and 2b respectively.

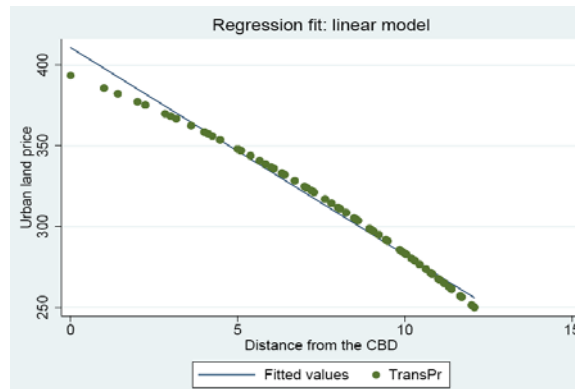


**Fig. 2.** Exp 1, Replication of the Alonso model (with homogeneous preferences for commuting)

The green area in Figure 2a represents agriculture and the black—the urban area. The intensity of grey colour in Figure 2b symbolizes the value of land: the darker the colour, the higher the land price. As in the benchmark case of a theoretical monocentric urban model, the land rent gradient is decreasing with distance. The urban land

price is equal for cells equidistant from the CBD. The city expansion stops at the location where the bid price of a buyer falls below the agricultural rent. The lightest-grey area in Figure 2b shows the beginning of agricultural area (urban-rural fringe) and symbolizes the city border. Note that not all of the buyers in the system ultimately purchase properties (only 465 of the 841 buyers engage in transactions). The parameter settings for Exp 1, then, replicate an open city model, where buyers are assumed to have the opportunity to purchase property in another location, if their bid price for available properties in this region is below the sellers' ask prices of the current land owners.

By applying a simple regression analysis to the model-generated data we estimated a land-price gradient<sup>7</sup> (Table 3 and Figure 3). Figure 3 shows a down-sloping demand function for land in the simulated urban zone. The horizontal axis shows the distance from the CBD in spatial units (a spatial unit can be interpreted as 1 km or 1 mile, although in our generalized analysis we do not refer to any specific unit here). The vertical axis shows the urban land price in monetary units (such as dollars or euro).



**Fig. 3.** Land rent gradients for Exp 1, linear regression fit of the model-generated data. *TransPr* – actual land transaction prices, *Fitted value* – estimated land rent gradient

### 3.2 Experiment 2

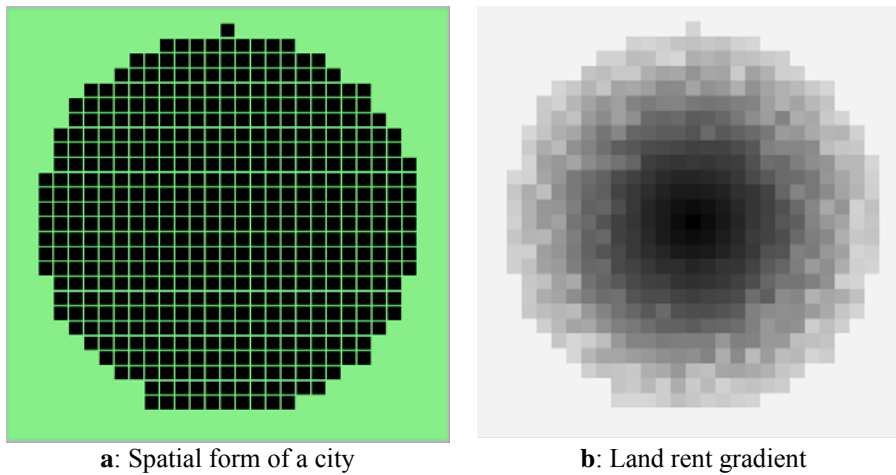
The setup in this experiment is almost that same as in Exp 1 (see Table 1) except for the fact that agents are heterogeneous with respect to preference for proximity. We assume that agents have different tolerance for commuting, i.e.  $\beta$  from the utility function follows uniform distribution in the range  $[0.7; 1]$  with mean equal to 0.85<sup>8</sup>. Analytical calculation of an equilibrium land price is possible only with homogeneous preferences.

<sup>7</sup> The results of linear regression model showed the best fit. The  $R^2$  values for linear, log-log, semi-log and inverse semi-log functional forms were 0.9923, 0.8166, 0.9738 and 0.8647 respectively.

<sup>8</sup> We also run the model with the normal distribution of preferences but the results are not presented in this paper because of the paper length limitations.

The first difference from Exp 1 manifests itself in the spatial morphology of the city as can be seen from comparison of Figures 2a and 4a. The city border has expanded a bit and urban population has increased (from 465 to 507) as can be seen in Table 2. Thus, if agents have different tolerance levels for commuting and are not constrained by the remoteness of the location (e.g., use private cars instead of public transport) then this is already enough to cause the urban area to sprawl even if green amenities are distributed homogeneously across the city. Essentially, heterogeneity in individual preferences for proximity may be a contributor to urban sprawl.

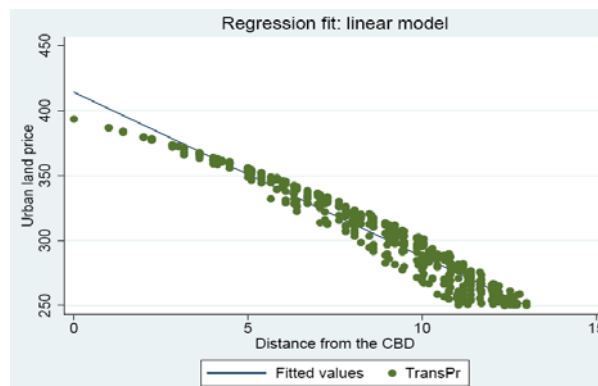
Additionally, the city no longer expands uniformly in all directions (compare the south, north, west and east borders of the city on Figure 4a). Since people have different preferences for proximity, there are just a few individuals tolerant enough to commuting to locate at the most distant edges of the city, such as a person in the north of the city (Figure 4a).



**Fig. 4.** Exp 2, Monocentric urban model with heterogeneous agents (with respect to commuting preferences)

The land price gradient (Figure 4b) is decreasing with distance as in Exp 1 (Figure 2b). However, in Exp 2 the prices of the cells at the same distance from the CBD are no longer equal because of preference heterogeneity. Thus, neither the spatial form nor land prices are symmetric in the city with heterogeneous agents. The average bid price is slightly higher in the city with heterogeneous preferences for proximity to CBD than in the homogeneous case; the total property value is about 9% higher in Exp 2 than in Exp 1 (see Table 2). On the one hand, agents with higher than average preferences for proximity, i.e.  $\beta > 0.85$ , bid more for the urban land that is closer to the CBD than an average agent, i.e.  $\beta = 0.85$  as in Exp 1. On the other hand, agents with lower than average preferences for proximity, i.e.  $\beta < 0.85$ , bid more for the remote spatial good than an average agent from Exp 1 would do, because the former are more tolerant to commuting. In Exp 2 the average  $\beta$  of the agents who actually settled in the city is 0.79, meaning that agents more tolerant to commuting out bid agents with strong preferences for proximity to CBD.

The estimated land rent gradient of the computer-generated data from Exp 2 is presented in Figure 5 and in Table 3. From Figure 3 it can be seen that transaction data are almost on the regression line. In contrast, data on transaction land price from Exp 2 is more dispersed, but is still down sloping as in the Alonso bid rent theory. The dispersion (essentially, distance-dependent heteroskedasticity) arises from the preference heterogeneity. Standard econometric theory also tells us that this rent gradient estimate is biased due to the omitted variable of preference heterogeneity. This simple modelling exercise illustrates that observed variation in real-world transaction prices may arise from non-spatially-uniformly distributed unobserved agent-level characteristics rather than from unbiased random error. Thus, rent gradient estimates that do not control for agent-level heterogeneity are likely to be systematically biased.



**Fig. 5.** Land rent gradients for Exp 2-1. linear regression fit of the computer generated data. *TransPr* – actual land transaction prices, *Fitted value* – estimated land rent gradient

The explained variation ( $R^2$ ) from Exp 1 is higher in Exp 1 than Exp 2-1 (0.9905 vs. 0.9560, Table 3). The settings for Exp 1 are very abstract, especially in its assumption of homogeneous preferences for location. If everybody in the city behaves as a *representative agent* does then *land prices can be fully explained only by the characteristics of the spatial environment*, such as distance. In practice, this is the only information usually available for hedonic price estimation. However, in reality agents preferences for the spatial good vary. Therefore, the percent of variation of the land price explained by land characteristics only decreases. The ABM environment allows us to link information about agents' preferences to transaction data and analyze it in the regression analyses. We re-ran the regression model using data from Exp 2, including agents' preferences for proximity as a second independent variable (see Table 3, column 4). As expected, the explained variation in land price increases (i.e.  $R^2$  is 0.9832 instead of 0.9560). Further, the estimated coefficient on distance to CBD declines in absolute value from 12.64 to 11.98, indicating that the first model overestimated the influence of transportation costs on hedonic land values.

Certainly, data about individual preferences is not easily acquired. Usually this requires a survey or role-playing games. Such micro level data can be used to construct empirical ABMs. ABMs fed with empirical data at the micro decision level [14, 17] provide nice examples of analysis of both the *spatial* and *agent-level drivers* of land-use change. Thus, an ABM land market supported by micro-level data on preferences

for location can serve as a computational laboratory in which we have a full understanding of the agent-level and spatial factors that influence *bid prices*, *ask prices*, and *realized transactions*.

## 4 Conclusions and discussions

In this paper, we have presented an agent-based land market model and analyzed the macro outcomes of simulations for the case of homogeneous and heterogeneous agents' preferences. In our ABM market, there is no one unique equilibrium-determined price for everyone in the market; rather, there is a set of individual transaction prices determined via bilateral trading by each set of trading partners separately. In spite of the fact that the centralized price determination mechanism is replaced by the distributed bilateral trading, the ALMA model with homogeneous agents reproduces the qualitative results of a monocentric urban model.

In the case of heterogeneous individual preferences for proximity, the land price gradients no longer exactly follow the predictions of the analytical model. In particular, the land price still generally decreases with distance to the city centre, but the prices of the equidistant cells are no longer equal since individuals with heterogeneous tolerance for commuting value them differently.

The most interesting result is that the city border has expanded due solely to the introduction of heterogeneity in agents' preferences for proximity. Essentially, the existing of the agents who are more tolerant for commuting creates a ground for urban sprawl. Thus, heterogeneity among individual location preferences is likely to be one of the factors causing urban sprawl. To our knowledge this result has not been reported before. Empirical econometric modelling [18, 19] has demonstrated the relationship between urban sprawl and landscape heterogeneity (green amenities). Agent-based urban models demonstrate that heterogeneous agents and heterogeneous landscape in combination exacerbate urban sprawl [14]. However, the fact that heterogeneity in preferences per se causes city expansion and spatially heterogeneous land rent patterns is a new result that could be demonstrated only through the agent-based land market, since the standard urban economic models cannot be solved with heterogeneous preferences.

The introduction of preference for open-space amenities and/or aversion to urban density has been shown to produce discontinuous patterns of development [14, 20-22]. We expect similar results with the ALMA model when open-space amenities are introduced. We also expect that the combination of heterogeneity of preferences for proximity and a heterogeneous landscape will exacerbate urban expansion and sprawl, especially if the distribution of open-space amenities is modelled in a realistic way. Usually, the level of environmental amenities increases with distance from the CBD. So, those people who are already tolerant for commuting receive additional benefits of settling farther from the city centre. These households are willing to pay more for a remote location if it has a scenic view or a park close by, so more open space is converted into urban use and the city expands further. We leave these experiments for future work.

With the help of a simple regression analysis of the model-generated data, we demonstrated that the data on individual preferences (available in the case of ABM) increases the explained variation in land prices. Essentially, we have created a computational laboratory in which we have a full understanding of the agent-level and spatial factors that influence bid prices, ask prices, and realized transactions. This laboratory lets us explore the statistical predictions that emerge from these models, creating an opportunity for greater understanding of the potential processes that have generated the transaction data that we observe in the real world.

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