GEOG524/EVPP524 Fall 2008: Utility and Consumer Demand*

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1 Learning objectives for this week

The main goal for this week is for you to understand:

- What a demand curve is;
- How the demand curve is affected by:
  - Agent preferences (Utility and the marginal rate of substitution),
  - Agent resources (Income and the budget constraint),
  - Relative prices of goods consumed.

2 Utility and Properties of Preferences

- **Utility** *Utility* is defined as "the pleasure or satisfaction that people get from their economic activity" (Nicholson, p. 55) Some important points: 1) In models of utility, and in fact models in general, economists invoke the *certeris parabus* assumption, which means that all other economic choices and resources are assumed to be held constant. We will note some examples of this. 2) Utility itself is not directly measurable or comparable across individuals. (Some argue, then, that the "theory" of utility is not a good one, since its predictions are not always testable.) 3) Utility theory implies only an ordinal ranking of choices. The utility model is based on some basic assumptions about preferences:

  1. **Complete Preferences** An individual is able to rank bundles of goods and state which one is preferred.
  2. **Transitivity** If A is preferred to B, and B is preferred to C, then A is preferred to C.

 All *definitions* from Nicholson 2004 unless otherwise stated.

3. **"Goods" are good** More is preferred to less. (See figure 2.1) Note also that there is such a thing as an economic "bad"; see figure 2.5 b. What are some examples of economic bads relevant for this class?

- **Indifference Curves** An *indifference curve* shows all combinations of goods and services that provide the same level of utility. Let's say we have a utility function with this specific form[1]:

$$u(x, y) = xy^2 \tag{1}$$

You can imagine that the values of x and y represent coordinates on a map, and the level of utility tells us the height of a hill at that location. Since our map is flat, we represent the height of the utility "hill" through an indifference curve. Everywhere along the indifference curve, the level of utility will be the same. Mathematically, for the particular utility function used here, the indifference curve is defined by:

$$u(x, y) = xy^2 \equiv \bar{U} \tag{2}$$

The analogy on a map is a topographic line. All points to the Northeast offer a higher level of utility (remember, more is preferred to less). We want to know the slope of this line in the x, y plane.

Question: Based our assumptions about utility, why can't indifference curves cross?

- **Marginal rate of Substitution** The **MRS**, or the *marginal rate of substitution* is the (absolute value of the) slope of an indifference curve. (For the mathematically inclined, the slope is given by $\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}}$) In a two good world, it answers the question *"How much of one good would I be willing to give up to get another unit of the other good?"* In a sense the MRS is like an internal opportunity cost, although we will use that term instead for the rate at which goods can be traded in the market. Note that this internal willingness to trade is not always constant along the indifference curve.

- **Diminishing Marginal Rate of Substitution** In our example, the MRS is $\frac{y}{2x}$. This consumer's preferences, then exhibit *diminishing marginal utility* for both goods, and therefore the indifference curves shows a diminishing marginal rate of substitution. *Marginal Utility* is the extra utility that a consumer gets by consumer one more unit of a good. Diminishing MU and MRS imply that consumers prefer balanced bundles of goods to unbalanced ones, or that more simply, as they get a lot of one thing and not as much of something else, they value the first thing less relative to the second. (For the mathematically inclined, this assumption implies that indifference curves will be strictly convex to the origin.) Examples of other sorts of preferences (useful and interesting, but too in depth for this class) are given in the book.

Question: What would it mean if indifference curves were straight lines? What if they were concave to the origin?

[1] Many different equations can be used to represent a utility function, provided they have a set of mathematical properties that are consistent with the assumptions about preferences described above.

3 The Budget Constraint and Choice

We all understand (especially those of use who are in, or have been in, graduate school) that our economic choices are constrained by our available resources. In our consumer model, the constraint on available resources is represented by a *budget constraint.* It shows the combinations of goods and services that a consumer can afford, given his or her income. In general it is simply represented by a straight line. The equation for this line simply says that the cost of each good times the quantity consumed adds up exactly to available income. If we graph the budget constraint, the area on the line and to the Southwest represents bundles of goods that the consumer can afford, and the area to the Northeast represents bundles that are not affordable. (*Question:* why would the consumer spend all her income, and what does this imply about where on the graph the optimal bundle will lie? In other words, what does this imply about the *shadow value* of income?)

$$p_x x + p_y y = I \;\Rightarrow\; y = \frac{I}{p_y} - \frac{p_x}{p_y}x \tag{3}$$

The slope of the budget constraint, $\frac{p_x}{p_y}$, represents the *opportunity cost* of consumption of a particular good for the consumer. It is the rate at which goods can be bought and sold in the marketplace.

Within this framework, we assume that consumers choose the bundle of goods that *maximizes their utility.* Graphically, the consumer will try to reach the highest indifference curve possible given her budget constraint. At this point, the slope of the budget constraint will equal the consumer's marginal rate of substitution:

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{p_x}{p_y} \tag{4}$$

Let's do a quick example to show how all of this works. An unnamed GMU faculty member, Dr. D. has \$60 to spend on entertainment. There are two activities she likes, performances at GMU's Center for the Arts (y) and ballroom dance classes (x). GMU performances are \$20 a ticket, and ballroom dance classes are \$10 a lesson. Her utility function for the two goods is the same as that given above in Equation **??**. What will she choose? First, graph her budget constraint. Note that the equation is:

$$10x + 20y = 60 \;\Rightarrow\; y = \frac{60}{20} - \frac{10}{20}x = 3 - \frac{1}{2}x \tag{5}$$

So, we know that the *opportunity cost*, or slope of the budget line is $-\frac{1}{2}$. This means that in the marketplace, the agent (Dr. D) can trade one unit of y for two units of x, since y is twice as expensive as x. This probably means that if each provided equal utility, she would probably choose more x and less y.

But, notice also something about her utility function. She gains relatively more utility from y than she does from x. (This might be because of something that is held fixed in our problem. What could it be?) So, if each event cost the same, she is likely to choose more y and less x. What would this imply about the shape of her indifference curves?

Remember, Dr. D has to chose a bundle that is feasible for her, which means it must be on her budget line. How about the bundle $(1, \frac{5}{2})$? First, let's compute her MRS at this point:

$$MRS = \frac{dy}{dx} = \frac{y}{2x} = \frac{\frac{5}{2}}{2} = \frac{5}{4}$$

This says that Dr. D would give up more than one unit of y to get another unit of x. Or in other words, to be willing to give up one more Center for the Arts performance, she would require $\frac{5}{4}$ of a ballroom dance class, in order to be as happy as she was before. However, the marketplace conditions imply that if she gives up one unit of y, she can get two units of x. What should she do? She should trade in some of her y for x and move down her budget constraint. She should continue to do this until the benefits of moving down the curve no longer outweigh the costs, or until she is indifferent between holding another unit of y and trading one in the marketplace. Alternatively, we notice that as she moves southeast from the point $(1, \frac{5}{2})$, along the budget line "road", she is climbing to a higher and higher level of utility – at least until she reaches the point of tangency. At this point, her internal willingness to trade will equal the rate at which trade occurs in the marketplace, or,

$$MRS = \frac{p_x}{p_y}$$

What point will this occur at? We can solve for this point using both our condition for optimal choice, Equation **??**, and the budget constraint, Equation **??**. First, solve for the tangency of the opportunity cost and MRS:

$$\frac{y}{2x} = \frac{1}{2} \Rightarrow \ y = x \tag{6}$$

Plugging this into the budget constraint:

$$y = 3 - \frac{1}{2}y \Rightarrow \ \frac{3}{2}y = 3 \Rightarrow \ y = 2 \tag{7}$$

Plugging this result back into Equation **??**, we get the optimal bundle (2,2): two GMU performances and two ballroom dance classes.

4 Changes in price and the demand curve

Let's say that due to budget cuts GMU has to end the half-price season ticket subsidy for faculty. Therefore, the price of GMU tickets rises to \$40 a ticket. How will this change Dr. D's budget constraint? The new constraint is:

$$y = \frac{60}{40} - \frac{10}{40}x = \frac{3}{2} - \frac{1}{4}x \tag{8}$$

How will this change Dr. D's optimal choice? Using the new price ratio:

$$\frac{y}{2x} = \frac{10}{40} = \frac{1}{4} \Rightarrow \ y = \frac{1}{2}x \tag{9}$$

Plugging into the *new* budget constraint:

$$\frac{1}{2}x = \frac{3}{2} - \frac{1}{4}x \Rightarrow \ x = 2 \tag{10}$$

Using Equation **??**, $y = 1$. (This meets her budget constraint–check and make sure!) The important point here is that as the price of GMU performances has increased, Dr. D. chooses to attend fewer. In fact what we are doing in this exercise is demonstrating the *demand curve*, which tells us how much of a particular good Dr. D will choose as the price of that good changes, holding everything else fixed. Our exercise illustrates one of the key implications of consumer choice theory–that demand curves are downward-sloping. (*Question*: What does this imply about the qualitative relationship between the price of a good and the quantity consumed of that good? Does this makes sense to you?)

As an exercise, see how Dr. D's choice of GMU performances would change if their price were to fall to $10 a ticket. Then, plot out the three solutions that we have obtained with the price of GMU performances on the Y axis and the quantity of GMU performances she chooses on the X axis. This is her demand curve for GMU performances.

5 Changes in Income

Let's say that Dr. D writes a successful grant proposal and obtains summer research money. As a result, she decides that she can spend twice as much on high-end entertainment[2]. Her new budget constraint will be:

$$10x + 20y = 120 \ \Rightarrow \ y = \frac{120}{20} - \frac{10}{20}x = 6 - \frac{1}{2}x \tag{11}$$

Graphically, what has happened to the budget constraint? What will be her new optimal choice for ticket purchases? (I'll leave this to you to do as an exercise).

[2]Her time budget was not included in this problem, obviously!