1 An exercise: Tax problems revisited

Now, you are going to get a chance to work through the tax example we did in class and convince yourself that consumers with "well behaved" preferences will be better off under an income tax than a sales tax. Here's our story: Chris' utility function is given by:[1]

$$u(x_1, x_2) = x_1 x_2$$

Currently, she faces the following prices and has the following income:

$$\begin{aligned} p_1 &= 1 \\ p_2 &= 1 \\ M &= 4 \end{aligned}$$

You need to determine which of these two cases she would be happiest under:

1. She faces a per-unit sales tax on good one of \$1 (i.e., $t_1 = 1$).
2. She faces an income tax equal to the revenue that the government would receive from the sales tax in part 1 (i.e., $T = R = t_1 x_1^n$)

Hints: Remember that "happiest" means the same thing as "highest level of utility". So what you want to do it compare the utility levels achieved under each scenario. I would follow the following steps:

1. First, solve for the general form of the demand functions for the original problem, in term of the *parameters* of the problem, p_1, p_2, M.
2. Next, carefully set up the budget constraint for scenario 1, adding in the per-unit tax. What can you extrapolate about the price that the consumer will now "see" for good one? Can you express this as a new price?

   Now, use the modified parameters of the problem to solve out for Chris' demands for the two goods and her resulting level of utility. Given her demands, how much revenue will the government earn, assuming that she is the only citizen?
3. Now, carefully set up the budget constraint when Chris faces an income tax. Assume that the amount of the income tax is exactly the revenue that the government would have earned from the tax in part 2. What is Chris' new disposable income? Can you express this as a new income parameter?

   Once again, use the modified parameters of the problem to solve out for Chris' demands and level of utility.
4. Compare the level of utility that Chris can achieve under cases 1 and 2. Which would she prefer? Compare the government's well-being in each case. Which case would the government prefer? Is case 1 Pareto optimal? Why or why not?
5. Express your solution graphically, labeling each case carefully.

[1] What utility function is this and what will the indifference curves look like?

2 The answer

Solving step by step:

1. The general solution for the demand functions in this case is:

$$x_1^n = \frac{M}{2p_1}$$
$$x_2^n = \frac{M}{2p_2}$$

For details on this solution, see Varian, p. 93, the answers for tutorial #3, or your class notes.

Just for reference, if Chris faces no taxes, her demands are:

$$x_1^n = \frac{4}{2*1} = 2$$
$$x_2^n = \frac{4}{2*1} = 2$$

and her utility level is:[2]

$$U_n(x_1^n, x_2^n) = 2 * 2 = 4$$

2. If Chris faces a per-unit tax, we write her new budget constraint as:

$$p_1 x_1 + t_1 x_1 + p_2 x_2 = M$$

or, rearranging:

$$\underbrace{p_1 + t_1}_{p_1'} x_1 + p_2 x_2 = M$$

Notice how we have incorporated the sales tax into a new price parameter, $p_1' = p_1 + t_1 = 2$. Solving for her demands:[3]

$$x_1^t = \frac{M}{2p_1'} = \frac{4}{2*2} = 1$$
$$x_2^t = \frac{M}{2p_2} = \frac{4}{2*1} = 2$$

Her utility level is:

$$U_t(x_1^t, x_2^t) = 1 * 2 = 2$$

The total revenue that the government collects is:

$$R = t_1 x_1^t = \$1 * 1 = \$1$$

3. If Chris faces an income tax, we can rewrite her budget constraint as:

$$p_1 x_1 + p_2 x_2 = \underbrace{M - R}_{M'}$$

[2] I'm using the subscript n to denote the no-tax case.

[3] I'm using the subscript t to denote the sales tax case.

Notice how we have expressed her new disposable income in terms of the parameter $M' = M - R = 4 - 1 = 3$. Using this new parameter to find her optimal demands:[4]

$$x_1^T = \frac{M'}{2p_1} = \frac{3}{2*1} = \frac{3}{2}$$
$$x_2^T = \frac{M'}{2p_2} = \frac{3}{2*1} = \frac{3}{2}$$

Her utility level is:

$$U_T(x_1^T, x_2^T) = \frac{3}{2} * \frac{3}{2} = \frac{9}{4}$$

4. Chris' utility level under the income tax is higher than under the sales tax. Of course, she would prefer no tax at all:

$$U_n = 4 > U_T = \frac{9}{4} > U_t = 2$$

The government earns \$1 by using either the sales tax or the income tax, so it is indifferent between the two tax instruments. Case 1, the sales tax case, is **not** Pareto optimal, because we could make Chris better off without making the government worse off. Case 2 looks like it could be Pareto optimal, but we would have to go through graduate school to be sure.

[4] I'm using the subscript T to denote the income tax case.