GEOG524/EVPP524 Fall 2004: Optimal Abatement and Tradeable Permits*

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1 Allocative and cost efficiency in pollution control

Previously, we've look at pollution that is assumed to be directly linked to a production process, and have discussed how Pigovian taxes and Coasean bargaining can in principle lead to a socially optimal level of production, and therefore pollution. This level is in theory *allocatively efficient*, since it implies that resources are devoted to their socially efficient use.

This is example is a bit simplistic in the world of complex technology that we live in, however. The reason is that pollution mitigation and cleanup technologies (pollution abatement) are available, so that firms may be producting the same levels of output, but one firm may be polluting less than another. Today we are going to focus on the idea of determining the appropriate level of pollution abatement (*allocative efficiency*) and having that pollution target achieved at least cost (cost efficiency). We will also note that the two concepts will not necessarily go hand in hand.

2 The Market Marginal Abatement Cost Curve

Most industrties that pollute contain firms which are heterogenous with respect to how costly it is for them to abate pollution. Older firms may have to incur higher costs, and newer firms may have lower costs, for example. For simplicity, let's look at a market with only two firms. Their marginal abatement cost curves are given by:

$$MAC_1 = 3A_1 \tag{1}$$

$$MAC_2 = \frac{3}{2}A_2 \tag{2}$$

where A_1 and A_2 are the abatement levels of each firm, respectively, and MAC_1 and MAC_2 are the marginal costs of abatment up to a maximum of 10 units for firm 1 and 20

Note this is a version of the 2003 handout that illustrates the same concepts in terms of abatement, not emissions.

units for firm 2. *Question: How might you interpret the higher possible level of abatement for firm 2? What do the differences in the slopes of the abatement cost functions mean?* In short, the equations tell us the cost of cleaning up one more unit at the point where the firm has cleaned up A_i units of pollution.

In order to figure out the socially optimal level of total emissions, we want to set the aggregate marginal cost of cleanup equal to the aggregate marginal benefit. To do this we need an aggregate, or industry-wide, marginal abatement cost curve. Constructing the aggregate marginal cost curve is similar, in principle, to constructing the *private* market supply curve. The equation needs to answer the question "For any given marginal cost of abatement, what level of abatement can be provided by each firm?" This means that we have to find the level of abatement for each firm, in terms of marginal cost. To find this, solve the given marginal cost functions for the levels of abatement:

$$\begin{aligned} A_1 &= \frac{1}{3} MAC_1 \quad (3) \\ A_2 &= \frac{2}{3} MAC_2 \quad (4) \end{aligned}$$

Now, we want to know what the total level of abatement would be for a given abatement cost, so we assume that the values of MAC_1 and MAC_2 are equal. As well, the *total* level of abatement is given by:

$$A = A_1 + A_2$$

Substituting in from above, we get:

$$A = A_1 + A_2 = \frac{1}{3} MAC_1 + \frac{2}{3} MAC_2 = MAC$$

Solving for MAC, we get:

$$MAC = A$$

See Figure 1.

3 Free-market level of emissions

If the firm's don't have to, they won't spend any money on abatement. This implies that their marginal abatement costs will be zero. Recall that above I told you that firm 1 could abate a maximum of 10 units and firm 2 20 units. If we assume that technology would allow them to clean up all of their pollution, then we can further assume that in the absence of any regulation or voluntary compliance, the two firms will emit a total of 30 units of pollution.

4 The optimal level of abatement

Assume that pollution causes harm according to the following marginal damage function:

$$MDC = 60 - 2A$$

Notice also that we could use this function to figure out where social damages are zero. This point also occurs at 30 units of abatement. So again, we can assume either that 30 units of emissions are occuring, or that 30 units of *harmful* emissions are occuring.

The socially optimal level of abatement occurs where the marginal damage cost equals the marginal abatement cost. This is "allocative efficiency". (*Question:* Can you explain why other points would not be socially optimal?) To find this level, we solve for the intersection of the two curves. See figure 2. (*Question:* What does this imply about the socially optimal level of emissions?)

$$60 - 2A = A \Rightarrow A^{SO} = 20$$

5 Achieving allocative and cost efficiency through pptimal taxation

Similarly to the Pigovian taxes that we reviewed two weeks ago, I assert that a tax equal to the marginal damage cost at the optimal level of abatement could bring about the socially optimal level of abatement, because firms would have an incentive to clean up pollution as long as the cost of cleanup was lower than the tax rate. Above this level, firms would simply pay the tax and allow pollution to occur. We find the tax by plugging the optimal level of emissions in to the marginal damage function:

$$\text{Tax} = MAC(A^{SO}) = 20$$

What incentive will this tax have on the abatement decisions of each firm? See figure 3.1 for an illustration of Firm 1's incentives. Recall Firm 1 can only abate 10 units. At a tax of \$20 per unit of pollution, it could initially save money by cleaning up, since it costs less than \$20 to abate. For example, the cost for the first unit of abatement is $3 * 1 = 3$, \$17 less than paying the tax. It is easy to argue that the firm will abate up to the point where $t = MAC_1$. Solving:

$$3A = 20 \Rightarrow A_1^* = 6\frac{2}{3}$$

Similarly, to find the optimal level of abatement for firm 2, set $MAC_2 = t^*$:

$$\frac{3}{2}A = 20 \Rightarrow A_2^* = 13\frac{1}{3}$$

What total level of abatement does the tax achieve?

$$A^* = A_1^* + A_2^* = 6\frac{2}{3} + 13\frac{1}{3} = 20$$

So that tax has succeeded in achieving the allocatively efficient level of emissions. Figure 4 illustrates this solution, with total control costs, total damage costs, and total tax paid shown.

6 The cost efficient levels of emissions for each firm

We know we would like 20 units of abatement by both industries. We also know that the two firms are not equally efficient at controling pollution. Given this, what is the least expensive way to achieve our target of 20 units of abatement? Do the abatement levels chosen by the firms above achieve the needed abatement at least cost?

The *equimarginal principle of optimality* states that the marginal abatement costs should be the same across firms in order to abate at least cost. This is "cost efficiency". To convince ourselves that this condition makes sense, let's do a little thought experiment. In order to do this experiment, it helps to graph the marginal abatement cost functions of both firm in a box graph, with firm 1 on the left and firm 2 on the right, assuming that between the two of them they abate the optimal 20 units. (See figure 5)

First, let's start by assuming that each firm is required to reduce abatement equally, so that each firm abates 10 units. At that level, each firm's marginal abatement costs would be

$$\begin{aligned} MAC_1 &= 3 * 10 = 30 && (5) \\ MAC_2 &= \frac{3}{2} * 10 = 15 && (6) \end{aligned}$$

and (using calculus) each firm's total abatement costs would be:

$$\begin{aligned} TAC_1 &= \frac{3}{2}A_1^2 = \frac{3}{2}(10)^2 = 150 && (7) \\ TAC_2 &= \frac{3}{4}A_2^2 = \frac{3}{4}(10)^2 = 75 && (8) \\ TAC &= 225.5 && (9) \end{aligned}$$

Notice that is is cheaper for Firm 2 to clean up abatement at this point than Firm 1. So, what if we had Firm 2 abate by one more unit, and allowed Firm 1 to emit one more unit? Now, their marginal abatement costs would be:

$$\begin{aligned} MAC_1 &= 3 * 9 = 27 && (10) \\ MAC_2 &= \frac{3}{2} * 11 = 16.5 && (11) \end{aligned}$$

and their total abatement costs would be:

$$\begin{aligned} TAC_1 &= \frac{3}{2}A_1^2 = \frac{3}{2}(9)^2 = 121.5 && (12) \\ TAC_2 &= \frac{3}{4}A_2^2 = \frac{3}{4}(11)^2 = 90.75 && (13) \\ TAC &= 212.25 && (14) \end{aligned}$$

Note, that the firms' marginal abatement costs are getting closer together, and total abatement costs have gone down. We are getting the same level of cleanup at lower cost.

It can be shown (using calculus) that the minimum abatement cost will occur when the marginal abatement costs of the two firms are equal. In order to find the optimal levels of cleanup for both firms, we set the marginal abatement costs equal, and we also use the fact that the sum of abatement from each firms has to equal the optimal level, A^*:

$$\underbrace{3A_1}_{MAC_1} = \underbrace{\frac{3}{2}A_2}_{MAC_2} \Rightarrow A_1 = \frac{1}{2}A_2 \tag{15}$$

$$A_1 + A_2 = 20 \tag{16}$$

Solving, we obtain:

$$A_1^{SO} = 6\frac{2}{3}; \quad A_2^{SO} = 13\frac{1}{3}$$

Notice that it is optimal for firm 1 to abate less. This is due to the fact that it costs more for firm 1 to clean up than firm 2. This solution is pretty close to the last example we tried. So we would expect total abatement costs to be close, but lower. (We would also expect them to be the same as those we found for tax from above–why?). Checking:

$$MAC_1 = 3 * (6\frac{2}{3}) = 20 \tag{17}$$

$$MAC_2 = \frac{3}{2} * (13\frac{1}{3}) = 20 \tag{18}$$

and their total abatement costs would be:

$$TAC_1 = \frac{3}{2}A_1^2 = \frac{3}{2}(6\frac{2}{3})^2 = 66.6667 \tag{19}$$

$$TAC_2 = \frac{3}{4}A_2^2 = \frac{3}{4}(13\frac{1}{3})^2 = 136.667 \tag{20}$$

$$TAC = 200 \tag{21}$$

Notice this is the same marginal and total abatement cost that we obtained with the tax. So, a tax of $ 20 for each unit of pollution would lead the two firms to emit the correct levels of abatement, and therefore achieve allocative efficiency at least cost.

Notice also that the uniform standard, which allowed each firm to pollute the same amount, was not cost efficient. Numerically we could caluculate the extra costs involved in the uniform stadard. We can also see this area on Figure 5. At 10 units of abatement each, Firm 1 cleans up at a marginal cost of 30, whereas it would cost Firm 2 only 15 to do that cleanup. Moving from $(10, 10)$ to the cost-efficent solution $(6\frac{2}{3}, 13\frac{1}{3})$, Firm 2 incurs cleanup costs equal to area A. It would have cost Firm 1 area A+B to do the same cleanup. So, if the two firms were allowed to bargain to change the division of emissions between the two firms, there are potential gains from trade equal to area $(A+B)-(A) = B$. *Question: What are the marginal gains from trade for the first unit of trading?* Once again, we would not expect that entire amount to be paid to firm 2 to do the additional abatement. As well, we would expect that firm one would be compensated something above their costs of abatement for the trade. The agreement that is struck may depend on the relative bargaining power of the two firms.

7 Allocative vs. cost efficiency

In this example, we have achieved both allocative and cost efficiency. In the real world, one may occur without the other. The government might set the right standard, but then impose (for example) a uniform division of pollution allowances and not allow trading. Or, a standard may have been set according to some other criteria, but permit trading might (in theory) lead to cost efficiency. In today's regulatory environment, there is a lot of emphasis on achieving cost efficiency, but less on setting an allocatively efficient standard.