GEOG590-002/EVPP741-002 Fall 2003: Optimal Emissions and Tradeable Permits*

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1 Allocative and cost efficiency in pollution control

Previously, we've look at pollution that is assumed to be directly linked to a production process, and have discussed how Pigovian taxes and Coasean bargaining can in principle lead to a socially optimal level of production, and therefore pollution. This level is in theory *allocatively efficient*, since it implies that resources are devoted to their socially efficient use.

This is example is however a bit simplistic in the world of complex technology that we live in, however. The reason is that pollution mitigation and cleanup technologies are available, so that firms may be producting the same levels of output, but one firm may be polluting less than another. Today we are going to focus on the idea of determining the appropriate level of pollution (*allocative efficiency*) and having that pollution target achieved at least cost (cost efficiency). We will also note that the two concepts will not necessarily go hand in hand.

2 The Market Marginal Control Cost Curve

Most industrties that pollute contain firms which are heterogenous with respect to how costly it is for them to control pollution. Older firms may have to incur higher costs, and newer firms may have lower costs, for example. For simplicity, let's look at a market with only two firms. Their marginal control cost curves are given by:

$$MCC_1 = 30 - 3e_1 \quad (1)$$

$$MCC_2 = 30 - \frac{3}{2}e_2 \quad (2)$$

where e_1 and e_2 are the emissions level of each firm, respectively, and MCC_1 and MCC_2 are the marginal costs of cleanup for that level of emissions. In short, the equations tell us the cost of cleaning up one more unit at the point where the firm only emits e units of pollution.

In order to figure out the socially optimal level of total emissions, we want to set the aggregate marginal cost of cleanup equal to the aggregate marginal benefit. (We did this in class two weeks ago.) To do this we need an aggregate, or industry-wide, marginal cost curve. Constructing the aggregate marginal cost curve is similar, in principle, to constructing the *private* market demand curve. The equation needs to answer the question "For any given marginal cost of cleanup, what level of pollution will occur?" This means that we have to find the level of pollution for each firm, in terms of marginal cost. To find this, solve the given marginal cost functions for the levels of pollution:

$$e_1 = 10 - \frac{1}{3}MCC_1 \quad (3)$$

$$e_2 = 20 - \frac{2}{3}MCC_2 \quad (4)$$

Now, we want to know what the total level of pollution would be for a given control cost, so we assume that the values of MCC_1 and MCC_2 are equal. As well, the *total* level of pollution is given by:

$$E = e_1 + e_2$$

Substituting in from above, we get:

$$E = e_1 + e_2 = (10 - \frac{1}{3}MCC) + (20 - \frac{2}{3}MCC)$$

Solving for MCC, we get:

$$MCC = 30 - E$$

which, to my great suprise, is the same aggregate marginal control cost function that I used in class two weeks ago!

3 Free-market level of emissions

If the firm's don't have to, they won't spend any money on abatement. This implies that their marginal control costs will be zero. You can imagine that the marginal control cost curves were constructed by starting at this point, and then asking how much it would cost for each firm to clean up one unit. To find the free-market pollution levels, plug zero in to each marginal control costs function:

$$e_1^{FM} = 10 - \frac{1}{3} * 0 = 10 \quad (5)$$

$$e_2^{FM} = 20 - \frac{2}{3} * 0 = 20 \quad (6)$$

$$E^{FM} = e_1^{FM} + e_2^{FM} = 30 \quad (7)$$

In the absence of any regulation, the two firms will emit a total of 30 units of pollution.

4 The optimal level of pollution

Recall from two weeks ago that we assumed that pollution caused harm according to the following marginal damage function:

$$MDC = 2E$$

The socially optimal level of pollution occurs where the marginal damage cost equals the marginal control cost. This is "allocative efficiency". (*Question:* Can you explain why other points would not be socially optimal?) To find this level, we solve for the intersection of the two curves:

$$2E = 30 - E \Rightarrow E^{SO} = 10$$

as we found before. (*Question:* How much abatement from the free-market outcome is required to get to this point?) Recall also that we discussed how a tax equal to the marginal damage cost at each the optimal solution could bring about this solution, because the industry would have an incentive to clean up pollution as long as the cost of cleanup was lower than the tax rate. Above this level, firm would simply pay the tax and allow pollution to occur. Recall that we found the tax by plugging the optimal level of emissions in to the marginal damage function:

$$\text{Tax} = MDC(E^{SO}) = 2 * 10 = 20$$

5 The cost efficient levels of emissions for each firm

We know we would like only ten units of pollution by both industries. We also know that the two firms are not equally efficient at controling pollution. Given this, what is the least expensive way to achieve our target of 10 units of pollution?

The *equimarginal principle of optimality* states that the marginal control costs should be the same across firms in order to abate at least cost. This is "cost efficiency". To convince ourselves that this condition makes sense, let's do a little thought experiment. In order to do this experiment, it helps to graph the marginal control cost functions of both firm in a box graph, with firm 1 on the left and firm 2 on the right, assuming that between the two of them the emit only the optimal 10 units of pollution.

First, let's start by assuming that each firm is required to reduce pollution equally, so that each firm emits only 5 units of pollution. At that level, each firm's marginal control costs would be

$$MCC_1 = 30 - 3 * 5 = 15 \quad (8)$$

$$MCC_2 = 30 - \frac{3}{2} * 5 = 22.5 \quad (9)$$

and each firm's total control costs would be:

$$
\begin{aligned}
TCC_1 &= \frac{1}{2}(15)(10-5) = 37.5 && (10)\\
TCC_2 &= \frac{1}{2}(22.5)(20-5) = 168.75 && (11)\\
TCC &= 206.5 && (12)
\end{aligned}
$$

Notice that is is cheaper for Firm 1 to clean up pollution at this point than Firm 2. So, what if we had Firm 1 reduce emissions by one more unit, and allowed Firm 2 to emit one more unit? Now, their marginal control costs would be:

$$
\begin{aligned}
MCC_1 &= 30 - 3 * 4 = 18 && (13)\\
MCC_2 &= 30 - \frac{3}{2} * 6 = 21 && (14)
\end{aligned}
$$

and their total control costs would be:

$$
\begin{aligned}
TCC_1 &= \frac{1}{2}(18)(10-4) = 54 && (15)\\
TCC_2 &= \frac{1}{2}(21)(20-6) = 147 && (16)\\
TCC &= 201 && (17)
\end{aligned}
$$

Note, that the firms' marginal control costs are getting closer together, and total control costs have gone down. We are getting the same level of cleanup at lower cost. It can be shown (using calculus) that the minimum control cost will occur when the marginal control costs of the two firms are equal. In order to find the optimal levels of cleanup for both firms, we set the marginal control costs equal, and we also use the fact that the sum of emissions from each firms has to equal the optimal level, S^*:

$$
\begin{aligned}
\underbrace{30 - 3e_1}_{MCC_1} &= \underbrace{30 - \frac{3}{2}}_{MCC_2} \Rightarrow e_1 = \frac{1}{2}e_2 && (18)\\
e_1 + e_2 &= 10 && (19)
\end{aligned}
$$

Solving, we obtain:

$$
e_1^{SO} = 3\frac{1}{3}; \quad e_2^{SO} = 6\frac{2}{3}
$$

Notice that it is optimal for firm 1 to pollute less. This is due to the fact that it costs less for firm 1 to clean up than firm 2. This solution is pretty close to the last example we tried. So we would expect total control costs to be close, but lower. (We would also expect them to be the same as those we found for the tax two weeks ago–why?). Checking:

$$MCC_1 = 30 - 3 * (3\frac{1}{3}) = 20 \quad (20)$$
$$MCC_2 = 30 - \frac{3}{2} * (6\frac{2}{3}) = 20 \quad (21)$$

and their total control costs would be:

$$TCC_1 = \frac{1}{2}(20)(10 - (3\frac{1}{3}) = 66.6667 \quad (22)$$
$$TCC_2 = \frac{1}{2}(20)(20 - 6\frac{2}{3}) = 136.667 \quad (23)$$
$$TCC = 200 \quad (24)$$

Notice this is the same marginal and total control cost that we obtained with the tax. It's pretty easy to see, using the same graphical arguments that we did in the aggregate two weeks ago, that a tax of $ 20 for each unit of pollution would lead the two firms to emit the correct levels of pollution, and therefore achieve allocative efficiency at least cost.

Notice also that the uniform standard, which allowed each firm to pollute the same amount, was not cost efficient. Numerically we could caluculate the extra costs involved in the uniform stadard. We can also see this area on the graph. At 5 units of pollution each, Firm 2 cleans up at a marginal cost of 22.5, whereas it would cost Firm 1 only 15 to do that cleanup. Moving from $(5, 5)$ to the cost-efficent solution $(3\frac{1}{3}, 6\frac{2}{3})$, Firm 1 incurs cleanup costs equal to area A. It would have cost Firm 2 area B+C to do the same cleanup. So, if the two firms were allowed to bargain to change the division of emissions between the two firms, there are potential gains from trade equal to area $(A + B + C) - (A) = B + C$. Once again, we would not expect that entire amount to be paid to firm 1 to do the additional cleanup. As well, we would expect that firm one would be compensated something above their costs of cleanup for the trade. The agreement that is struck may depend on the relative bargaining power of the two firms.

6 Allocative vs. cost efficiency

In this example, we have achived both allocative and cost efficiency. In the real world, one may occur without the other. The government might set the right standard, but then impose (for example) a uniform division of pollution allowances and not allow trading. Or, a standard may have been set according to some other criteria, but permit trading might (in theory) lead to cost efficiency.