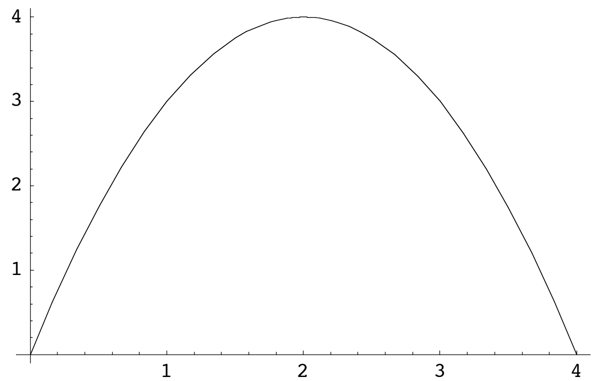
In[2]:=
```
(*Let's take a very simple function and review, or explore,
 how we can find the optimum (highest or lowest value) of that function*)
y = 4 x - x^2;
Plot[y, {x, 0, 4}, {AxesOrigin → {0, 0}}]
```

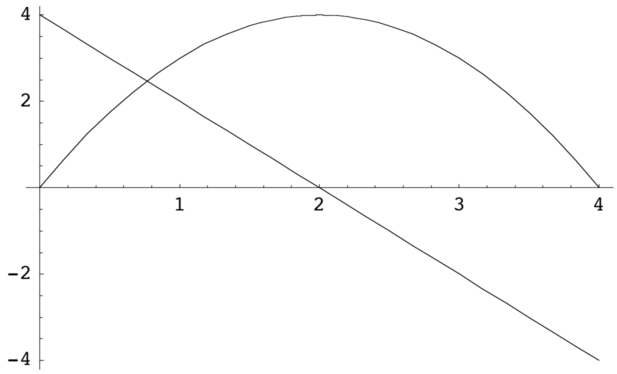


Out[3]= - Graphics -

In[4]:=
```
(*It's easy to see where the maximum of this function is,
 but how could we describe that mathematically?  A calculus-
  based description is based on the rate of change of the function.  Imagine
   if this were a hill that you were climbing.  It would be very,
 very steep at first, then less steep, then finally at the top,
 it would be flat.  The "steepness" of the function is,
 of course, its slope.  So in principle,
 we want to find the place where the slope is zero.  It's easy using
  calculus to find a function that tells us the slope of the tangent line:*)
slope = D[y, x]
```

Out[4]= 4 - 2 x

In[5]:=
```
(*Notice something if we plot the slope and the function on the same graph:*)
Plot[{y, slope}, {x, 0, 4}, {AxesOrigin → {0, 0}}]
```

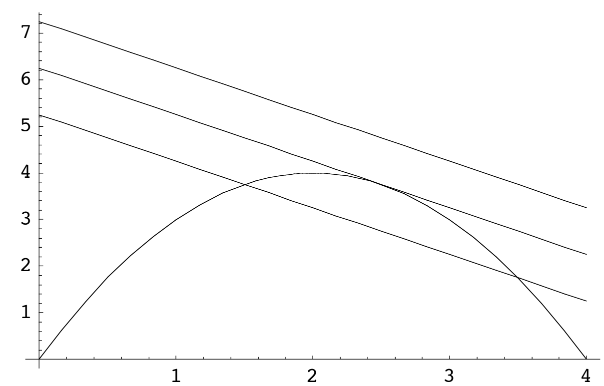


Out[5]= - Graphics -

```
(*Now, what we treat the function y as a "constraint".,
 and try to reach the highest value of another function,
 which we will call an "objective function."  This other function expresses
  a relationship between x and y also, but can take on different values, V,
 such that V = y + x.  Solve that for y and call it OF, then plot the function
  for a few different values of V.  Which do you think is the constrained optimum?*)
```

In[24]:=
```
OF = V - x;
```

In[39]:=
```
Plot[{y, OF /. V → (21 / 4), OF /. V → (25 / 4), OF /. V → (29 / 4)},
  {x, 0, 4}, {AxesOrigin → {0, 0}}]
```



Out[39]= - Graphics -

In[64]:=
```
(*Here' s how I solved for the optimal point.  First
  I solved for the optimal X by setting the slopes of the two
  functions equal.  Then I used that x to get the Y coordinate of the
  constraint.  Then I used that to solve for the intercept of the function,
 or the value of V.  I know the code seems crypic; blame Stephen Wolfram.*)
y = 4 x - x^2;
slope = D[y, x];
OF = V - x;
dOF = D[OF, x];
optx = Solve[dOF == slope, x][[1]]
optv = (y /. optx) + x /. optx
```

Out[68]= $\left\{x \to \frac{5}{2}\right\}$

Out[69]= $\frac{25}{4}$