

## Hotelling notes (from Hackett example)

Introduction to Environmental and Resource Economics, November 17, 2004

This is the example from the Hackett on-line lecture, hopefully a little easier to follow. The yearly demand for the resource is given by:

$$200 - Q_t$$

where  $Q$  is the quantity of the resource. The resource can be extracted at a constant marginal cost of \$10. (This is basically the supply curve.) There is a two-year time horizon, and a discount rate of 10%.

If the supply of the resource is not limited, the resource will be harvested to the point where supply equals demand, just like normal. Set demand = marginal cost:

$$200 - Q = 10 \Rightarrow Q = 190$$

If the supply is limited to 100 units in all, we can argue that it will be optimal for individual suppliers to save some of the resource for the second year, rather than extracting and selling it all in the first year. (Try plugging in the remaining 10 units for the second year into the price function, and notice how high the price would be in year 2 if there were only 10 units left, even if you discount.)

To find what we believe the market solution would be, solve for the set of allocations that equates the discounted marginal net benefits for the two years. Marginal net benefits are price (demand curve) less marginal cost. The MNB for any given year is:

$$200 - Q - 10 = 190 - Q$$

Marginal net benefits for the second year should be discounted back one year, by dividing by  $(1 + r) = 1.1$ . Setting the present discounted value of marginal net benefits equal across time periods:

$$190 - Q_0 = \frac{190 - Q_1}{1.1} \Rightarrow Q_1 = 1.1Q_0 - 19$$

Now, use the constraint on the resource ( $Q_0 + Q_1 = 100$ ) to solve for the exact allocation, by substituting in for  $Q_1$ :

$$Q_0 + (1.1Q_0 - 19) = 100 \Rightarrow Q_0 = 56.667; Q_1 = 43.333$$

The hotelling rent for the first year is:

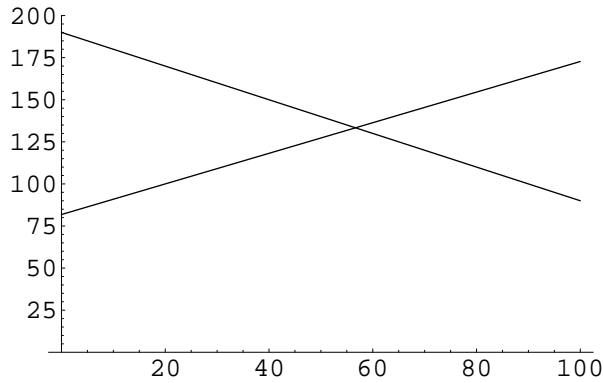
$$190 - 56.667 = 133.333$$

and for the second year:

$$190 - 43.333 = 146.667$$

The hotelling rent rises over time because the resource is becoming more scarce. As well, since returns are equal in discounted terms, returns have to be higher in absolute terms in future years. Consumption of the resource declines over time due to discounting (per Sara's presentation).

Here is a box graph (flawed, as before) that shows the two-period problem, and the optimal solution:



The downward-sloping function starting from the left axis is  $190 - Q_0$ . The function starting from the right axis is  $\frac{190 - Q_1}{1.1}$ . Notice that the  $Q_0$  function has a higher intercept (190) than the  $Q_1$  function, because the  $Q_1$  function is discounted. Notice also that the functions don't cross at an equal allocation. If you drew a line at 50,50, you could find the triangle of lost profit opportunities from the equal allocation—it would be the difference between the profits possible in the second time period and the profits gained in the first. Again, VERY similar to the tradable permits problems.