In[3]:=
```
(*Here is the original problem if the resource is non-renewable*)

mnb1 = 6 - Q1;
mnb2 = (6 - Q2) / (1 + r);
r = 0.10;
opt = Solve[mnb1 == mnb2, Q2][[1]]
lim = Q1 + Q2;
lim /. opt
q1 = Solve [(lim /. opt) == 10, Q1]
q2 = opt /. q1
```

Out[6]= {Q2 → 1.1 (-0.545455 + 1. Q1)}

Out[8]= Q1 + 1.1 (-0.545455 + 1. Q1)

Out[9]= {{Q1 → 5.04762}}

Out[10]= {{Q2 → 4.95238}}

In[11]:=
```
(*Now here is the Hotelling rent for each year,
 remember it is the difference between price and marginal cost*)
hr1 = 6 - Q1 /. q1
hr2 = 6 - Q2 /. q2
```

Out[11]= {0.952381}

Out[12]= {1.04762}

In[13]:=
```
(*Notice that the present value of hotelling rent is constant., so effectively,
 Hotelling' s rule says that profits grow at the rate of interest.  *)
pvhr1 = mnb1 /. q1
pvhr2 = mnb2 /. q2
```

Out[13]= {0.952381}

Out[14]= {0.952381}

In[15]:=
```
(*Now, let' s assume that the compost increased
  in productivity by 20 % a year.  Note this is a constant,
 not logistic growth rate.  Let' s call this "g".  Our marginal net benefit,
 or productivity, function stays the same.  But our constraint changes.*)
```

```
In[34]:= mp1 = 6 - C1;
         mp2 = (6 - C2) / (1 + r);
         r = 0.10;
         em = C2 - ((1 + g) (so - C1));
         so = 10;
         g = 0.20;
         opt = Solve[(mp1) / (1 + g) == mp2, C2][[1]]
         em /. opt
         c1 = Solve [(em /. opt) == 0, C1]
         c2 = opt /. c1

Out[40]= {C2 → 1.1 (5.45455 - 0.833333 (6. - 1. C1))}

Out[41]= 1.1 (5.45455 - 0.833333 (6. - 1. C1)) - 1.2 (10 - C1)

Out[42]= {{C1 → 5.43307}}

Out[43]= {{C2 → 5.48031}}

        (*Here is the new Hotelling rent for each year.  Notice the pure Hotelling rent
           falls over time ... the resource is becoming less scarce in this case.*)
        nhr1 = 6 - C1 /. c1
        nhr2 = 6 - C2 /. c2

Out[44]= {0.566929}

Out[45]= {0.519685}

In[46]:= (*What if you adjust for the growth rate and put them in present value terms?*)
         (*blech1 = (6 - C1 /. c1 ) (1.1)
           blech2 = (6-C2 /. c2) (1+g)*)
         npvhr1 = mp1 /. c1
         npvhr2 = (mp2 /. c2) (1 + g)
         (*So the lesson is that the growth rate of the resource
             counterbalances the effect of discounting ... this makes lots of intuitive
           sense.  The discount rate gives you an incentive to consume sooner,
          and the growth rate gives you an incentive to wait.*)

Out[46]= {0.566929}

Out[47]= {0.566929}

        (*What if the peat bog is open-acess?  Profits in year 1 will be zero:*)
        zp1 = Solve[(mp1) == 0, C1][[1]]
        Solve[(em /. zp1) == 0, C2][[1]]
        (*What is the marginal profit in each time period?  We know it is zero in the first
          time period.  Notice now the resource is more scarce in the second time period.*)
        6 -
          C2 /. %

Out[57]= {C1 → 6}

Out[58]= {C2 → 4.8}

Out[59]= 1.2
```