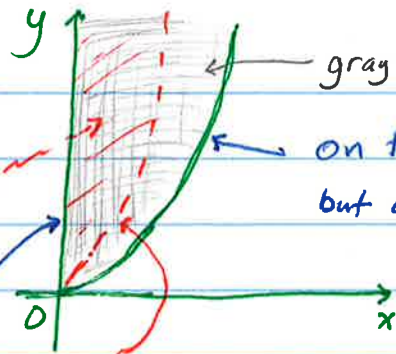
2(g)



gray: region where joint pdf is positive

on the boundary, $y = x^2$ and so $v = x^2/y = 1$

but above the boundary $y > x^2$ and so $v = x^2/y < 1$

on this boundary $x = 0$ and so $v = x^2/y = 0$

altogether, the support of $V = X^2/Y$ is $(0, 1)$

for value outside the support, $f_V(v) = 0$

for $v \in (0,1)$,

$$F_V(v) = P(X^2/Y \le v)$$
$$= P(Y \ge X^2/v)$$
$$= \int_0^\infty \int_{x^2/v}^\infty 2x e^{-y}\, dy\, dx$$
$$= \int_0^\infty 2x \left(\int_{x^2/v}^\infty e^{-y}\, dy \right) dx$$
$$= \int_0^\infty 2x\, e^{-x^2/v}\, dx$$
$$= -v e^{-x^2/v} \Big|_0^\infty$$
$$= v$$

alternatively this equals

$$P(X \le \sqrt{vY})$$
$$= \int_0^\infty \int_0^{\sqrt{vy}} 2x e^{-y}\, dx\, dy$$
$$= \int_0^\infty e^{-y} \left(\int_0^{\sqrt{vy}} 2x\, dx \right) dy$$
$$= \int_0^\infty e^{-y} \left(x^2 \Big|_0^{\sqrt{vy}} \right) dy$$
$$= v \underbrace{\int_0^\infty y e^{-y}\, dy}_{= \Gamma(2) = 1! = 1}$$
$$= v$$

&

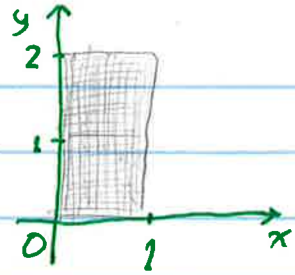
$$f_V(v) = \frac{d}{dv} v = 1$$

so altogether we have

$$\boxed{f_V(v) = I_{(0,1)}(v)}$$

For $v \in (0,1)$, this is $y = x^2/v$, and so need to integrate $f_{X,Y}(x,y)$ over this region to get $P(Y \ge X^2/v)$ (or equivalently, $P(X \le \sqrt{vY})$).

3(b)



The support of $W = \min\{X, Y\}$ is $(0,1)$.

For $w \in (0,1)$,

$$F_W(w) = P(W \le w)$$

$$= 1 - P(W > w)$$

$$= 1 - P(X > w, Y > w)$$

$$= 1 - \int_w^2 \int_w^1 \frac{6x^3 + 2xy}{5}\,dx\,dy$$

$$= 1 - \int_w^2 \left(\frac{3}{10}x^4 + \frac{yx^2}{5}\right)\Big|_w^1\,dy$$

$$= 1 - \int_w^2 \left(\frac{3}{10} + \frac{y}{5} - \frac{3w^4}{10} - \frac{yw^2}{5}\right)dy$$

$$= 1 - \left[\left(\frac{3}{10}y + \frac{y^2}{10} - \frac{3w^4y}{10} - \frac{y^2w^2}{10}\right)\Big|_w^2\right]$$

$$= 1 - \left[\frac{3}{5} + \frac{2}{5} - \frac{3}{5}w^4 - \frac{2}{5}w^2 - \frac{3}{10}w - \frac{w^2}{10} + \frac{3}{10}w^5 + \frac{w^4}{10}\right]$$

$$= \frac{3}{10}w + \frac{w^2}{2} + \frac{w^4}{2} - \frac{3}{10}w^5.$$

Altogether, we have

$$F_W(w) = \begin{cases} 1, & w \ge 1, \\ \dfrac{3w + 5w^2 + 5w^4 - 3w^5}{10}, & 0 < w < 1, \\ 0, & w \le 0. \end{cases}$$

Note: Plug 0 into here and you get 0, plug 1 into it and you get 1, so cdf goes from 0 to 1 over the support.

4(a) The support of $V = \max\{X, Y\}$ is $(0, 2)$, and so $f_V(v) = 0$ for $v \notin (0,2)$.

For $v \in (0, 2)$,

$$
\begin{aligned}
F_V(v) &= P(X \le v, Y \le v) \\
&\overset{\text{ind.}}{=} P(X \le v) P(Y \le v) \\
&= \left(\frac{v}{2}\right)\left(\int_0^v \frac{y}{2}\,dy\right) \\
&= \left(\frac{v}{2}\right)\left(\frac{v^2}{4}\right) \\
&= \frac{v^3}{8},
\end{aligned}
$$

& $f_V(v) = \frac{d}{dv}\left(\frac{v^3}{8}\right) = \frac{3}{8}v^2.$

Altogether, $\boxed{f_V(v) = \frac{3}{8}v^2 I_{(0,2)}(v)}.$