Midterm Exam 2

STAT 544, Spring 2020

Instructions: This is an open book and open notes exam; you can use any written or printed materials that you have brought with you, but you are not allowed to share any materials during the exam period. You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, about this exam during the exam period (and at most I will clarify the statement of a problem; I won't give you any hints about how to solve a problem). You can use a calculator and/or a computer if you wish to.

The 11 parts of the 5 problems are weighted equally.

Unless stated otherwise, for all parts of the exam you are to justify your answers, using notation and terminology properly, and clearly defining any events and random variables that you use (that aren't defined in the statement of the problem).

Express probabilities, expected values, variances, etc. as exact values (as fractions, integers, or in decimal form), or else round them to the nearest thousandth. Express them numerically — do not express final answers as expressions that need to be evaluated. (E.g., the standard normal cdf or the gamma function should not appear in any of your final answers.)

1) X is a random variable having pmf

$$p_X(x) = \frac{2^x}{7} I_{\{0,1,2\}}(x).$$

(a) Letting F_X denote the cdf of X, give the value of $F_X(1.5)$.

(b) Give the value of $E(X)$.

2) If X is a random variables for which $E(X) = 7$ and $Var(3X - 3) = 45$, what is the value of $E(X^2)$?

3) Suppose that a continuously operating machine malfunctions randomly, according to a Poisson process having a rate of 5 malfunctions per week (7 days).

(a) What is the probability that exactly 1 of the next 7 days has no malfunctions?

(b) What is the probability that the 1st malfunction of a week occurs sometime during the 2nd day of the week?

4) Consider a random variable X having cdf

$$F_X(x) = \begin{cases} 1, & x \geq 64, \\ \frac{x^{1/3}}{4}, & 0 < x < 64, \\ 0, & x \leq 0. \end{cases}$$

(a) Give the pdf of X.

(b) Give the value of $P(X \geq 27)$.

(c) Letting U be a uniform $(0, 1)$ random variable, give a function of U which has the same distribution as X.

(d) Give the cdf of $Y = X^{2/3}$.

(e) Give the value of $E(X^{2/3})$.

5) Letting X be a normal random variable having mean μ and variance σ^2, express the 49th percentile of X's distribution, $\xi_{0.49}$, as a linear function of μ and σ, rounding the coefficient of σ to the nearest thousandth. (*Note*: $\xi_{0.49}$ is a value that satisfies $P(X < \xi_{0.49}) = 0.49$ and $P(X > \xi_{0.49}) = 0.51$.)