Midterm Exam 1

Instructions: You cannot use any books or notes. You can use a calculator, but not a computer. You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period (and at most I will clarify a problem — I won't give you any help with solving the problems).

The exam consists of 5 problems, having a total of 6 parts. Each of the 6 parts is worth 20 points, and I'll count your best 5 of 6 scores from these parts to determine your overall exam score.

Express probabilities as exact values (as fractions, or in decimal form), or else round them to three significant digits. (Note: 0.00402 has three significant digits, and 0.004 only has one significant digit.) In order to receive full credit, do not express final answers as expressions that need to be evaluated. (So a final answer should not include a binomial coefficient or even a factorial.)

Be sure to *use notation and terminology properly!* For most of the parts which follow, I want you to *define events* (if they aren't already defined in the statement of the problem) and *provide justification* for your answers. (If a part can be solved just using Ch. 1 and Ch. 2 results, you don't necessarily have to define events, but when using Ch. 3 results you should define events.) Put all of your work on these sheets. If you need more room, direct me to look for additional work on the back of a page. **Draw boxes around your final answers!**

1) Suppose that an ordinary, fair, 6-sided die will be rolled three times. Making the usual assumptions about rolling a die multiple times, what is the probability that all three rolls will result in 6 spots on the upward face of the die, given that at least one of the three rolls will result in 6 spots on the upward face of the die?

2) Consider a bag of four tennis balls, three of which are brand new, and one of which has been used. Suppose that a ball is randomly selected from amongst the four balls. Then the ball is used and returned to the bag. (At this point the bag will either contain one or two used balls. If the randomly selected ball was new, it has now been used, and so in this case when it is returned the bag will contain two used balls and two new ones. On the other hand, if the randomly selected ball was the one used one, upon returning it to the bag the bag will contain only one used ball along with three new ones.) Finally, once the ball originally selected is returned to the bag, a second random selection will be made from the four balls in the bag.

(a) What is the probability that *both* random selections will result in a new ball? For this part, I definitely want you to define some events and make use of a simple result from Ch. 3 of the text.

(b) What is the probability that *the second* random selection will result in a new ball? (*Hint*: Noting that this would be trivial to answer if you knew whether the first ball randomly selected was new or used, does this fact suggest the use of a particular result from Ch. 3 of the text?)

3) Suppose a dash is transmitted with probability 0.6 and a dot is transmitted with probability 0.4. If a dash or dot is received correctly with probability 0.9 and received incorrectly (meaning a dash is received as a dot, or a dot is received as a dash) with probability 0.1, what is the probability that a dash is sent given that a dash is received?

4) Suppose that A, B, and C are events such that $P(A) = 0.3$, $P(B) = 0.2$, $P(C) = 0.1$, A and B are independent, B and C are mutually exclusive, and $C \subset A$. Give the value of $P(A^C \cap B^C \cap C^C)$. (*Hint*: Use one of DeMorgan's laws, along with the inclusion-exclusion result.)

5) Suppose that 3 men and 5 women will be randomly put into a linear arrangement, from left to right, with all 8! possibilities being equally likely? What is the probability that all of the women will be next to one another and all of the men will be next to one another?