Midterm Exam 1
STAT 544, Spring 2020

Instructions: This is an open book, open notes exam. You can use a calculator or computer if you wish to. You are not allowed to communicate with anyone about these problems (verbally, in writing, or electronically), except for me, during the exam period (and at most I will clarify a problem — I won't give you any help with solving the problems). You can spend as much time as you want as long as your e-mailed solutions get to me by 11 PM on Monday night (March 30).

There are 5 problems on this exam, having a total of 6 parts. All of the parts will be weighted equally. (When this was going to be an in-class exam, I was only going to count your best 5 of 6 scores. But for this take-home exam, all 6 parts will count.) To receive full credit, you need to **justify your answers**, using notation and terminology properly, and clearly defining any events that you use (that aren't defined in the statement of the problem).

Express probabilities as exact values (as fractions, or in decimal form), or else round them to three significant digits. (Note: 0.00402 has three significant digits, and 0.004 only has one significant digit.) Do not express final answers as expressions that need to be evaluated. (Your final answers should not include any letters, binomial coefficients, or factorials.)

You can either print out the exaam, write your solutions in the spaces provided, and then scan it and e-mail it back to me, or you can write or type your solutions on blank pages, and just send me your solutions. (If you do this, please be sure to have your solutions in the proper order (e.g., Problem 2 before Problem 3).) **Draw boxes around your final answers!**

1) To gather the items for a seafood omlet, suppose someone randomly chooses one piece of salmon from two pieces of salmon, one slice of cheese from three slices of cheese, and two eggs from four eggs. If one piece of salmon, one slice of cheese, and one egg have gone bad, what is the probability that there will be at least one bad item among the randomly chosen items? (Make a reasonable assumption of independence in order to obtain an answer, and be sure to note in your solution exactly where the assumption of independence is used.)

2) Suppose that 5 blocks, 1 with `A` on it, 2 with `I` on them, and 2 with `M` on them, are put in the order `MIAMI`. Then suppose that 2 of these 5 blocks are randomly selected and removed, and then they are put back into the same 2 locations they were taken from *in random order*. What is the probability that they are once again in the order `MIAMI`? (Note that if the 2 selected blocks happen to have the same letter on them, then however they are put back into the same two locations, the blocks will still spell `MIAMI`. But if the selected blocks have two different letters on them, the 5 blocks may or may not spell `MIAMI` after the 2 blocks are replaced in random order.)

3) If three fair 6-sided dice are rolled, what is the probability that all of them will result with 6 spots on their upward faces given that at least one of them will result with 6 spots on its upward face.

4) Suppose that 4 people are to be randomly given 3 balls each from a collection of 8 amber balls and 4 blue balls, with all possible ways of dividing the 12 balls into 4 groups of size 3 being equally likely. What is the probability that each of the 4 people will get a blue ball?

5) Suppose that 5 balls will be randomly dropped into a set of 3 buckets, labelled I, II, and III. (On each drop, each bucket is equally likely to get the ball, and the random selections of the buckets to get the balls are independent of one another.)

(a) What is the probability that each of the 3 buckets will contain at least one ball after the 5 balls are dropped into them?

(b) After all 5 balls have been dropped into the buckets, what is the probability that bucket I will contain at least three balls given that one of the buckets will contain at least three balls? (**Important comments**: One doesn't have to do part (a) in order to do this part. If you cannot give a short solution to this part, a longer solution is okay too (but a short solution is possible). For a short solution, don't *just* give an answer; adequate justification for the answer should also be given.)