STAT 652: HW #6

Instructions: Please present your solutions in order. As always, use paper which is approximately 8.5 inches by 11 inches, present tidy and easy-to-follow solutions, *draw boxes around your final answers* (but don't just give answers without supporting work), and staple all sheets together in the upper left hand corner. I don't like cover sheets, executive summaries, folders, binders, or paper clips.

You should **work the extra credit part entirely on your own**. You shouldn't discuss it with anyone else, get help from anyone else, etc.

1) X is a random variable having pdf

$$f_X(x|\theta) = \frac{\Gamma(2\theta)}{\Gamma(\theta)\,\Gamma(\theta)}\, x^{\theta-1}\,(1-x)^{\theta-1}\, I_{(0,\,1)}(x),$$

   where $\theta \in \Theta = \{1, 2\}$.
   (a) (8 points) Using X as a test statistic, give the rejection region for the MP size 0.1 test of $H_0 : \theta = 1$ vs. $H_1 : \theta = 2$.
   (b) (2 points) Give the power of the test requested in part (a).
   (c) (4 points) Considering the same test, what is the p-value if the one observes $x = 0.42$.
2) A ball is to be selected from an urn containing 5 balls, with all 5 balls having the same probability of being selected. Either the balls are numbered 1, 2, 3, 4, and 5, *or* all of the balls have the number 5 on them. Let X be the number on the selected ball.
   (a) (6 points) Give the critical function for the most powerful size 0.1 test (based on X) of the null hypothesis that the balls are numbered from 1 to 5 against the alternative that all of the balls are numbered with a 5.
   (b) (2 points) Give the power of the test requested in part (a). (*Note*: You don't have to give a power function since the alternative hypothesis is simple. Simply give the power of the test.)
3) (8 points) X_1 and X_2 are iid random variables having pdf

$$f(x) = \frac{2x}{\theta^2}\, I_{(0,\,\theta]}(x),$$

   where $\theta \in \Theta = (0, \infty)$. Consider testing $H_0 : \theta \geq 2.5$ vs. $H_1 : \theta < 2.5$ using a UMP test. Given the observed values $x_1 = 1.5$ and $x_2 = 1.3$, give the resulting p-value.
4) (8 points) Suppose that an urn contains 12 balls of θ different colors, with an equal number of balls of each color, where $\theta \in \Theta = \{2, 3, 4, 6\}$. Letting X be the number of colors obtained in a randomly drawn (without replacement) subset of size 3, give the test function (based on X) of a size 0.1 (generalized) likelihood ration test of $H_0 : \theta = 3$ vs. $H_1 : \theta \neq 3$.
5) (10 points) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x|\theta) = 4\theta x^3 e^{-\theta x^4}\, I_{(0,\,\infty)}(x),$$

   where $\theta \in \Theta = (0, \infty)$. Obtain an exact 99% confidence interval for θ which depends on the x_i only through the value of the complete sufficient statistic $\sum_{i=1}^n X_i^4$, and evaluate it for the case of $n = 10$ and $\sum x_i^4 = 23.6$, rounding each confidence bound to the nearest hundreth.
6) (8 *extra credit* points) X_1 and X_2 are iid random variables having pdf

$$f(x|\theta) = \frac{\theta}{x^2}\, I_{[\theta,\,\infty)}(x),$$

   where $\theta \in \Theta = (0, \infty)$. Obtain a level $1 - \alpha$ confidence interval for θ by inverting the acceptance region of a size α LRT of $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$, and supply the resulting interval estimate for the case of $\alpha = 0.01$, $x_1 = 6.62$, and $x_2 = 8.84$, rounding each confidence bound to the nearest hundredth.