STAT 652: HW #6

due 7:30 PM on May 9, 2006

(*Note*: There will be only a 15 minute grace period — not the usual two days.)

Instructions: Please present your solutions in order. As always, use paper which is approximately 8.5 inches by 11 inches, present tidy and easy-to-follow solutions, *draw boxes around your final answers* (but don't just give answers without supporting work), and staple all sheets together in the upper left hand corner. I don't like cover sheets, executive summaries, folders, binders, or paper clips.

I have included a way for you to partially check your work for some parts of this assignment. I'm willing to discuss the pertinent material presented in class with you, but *I do not plan to provide hints and suggestions about the specific parts of this homework.* (If you want to ask questions, make them general.)

1) (4 points) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x) = \frac{e^{-x/\theta}}{\sqrt{\pi\theta x}} I_{(0,\infty)}(x),$$

where $\theta \in \Theta = (0, \infty)$. Using a pivot based on the MLE of the scale parameter θ, obtain an exact 95% confidence interval for θ.

2) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x) = \theta x^{\theta-1}\, I_{(0,\,1)}(x),$$

where $\theta \in \Theta = (0,\, \infty)$.

(a) (4 points) Obtain a formula for an exact confidence interval for θ that depends on the data only through a complete minimal sufficient statistic, and has coverage probability $1 - \alpha$. Evaluate the confidence interval for the case of $\alpha = 0.1$, $n = 15$, and $\sum_{i=1}^{15} \log x_i = -17.94$, rounding each confidence bound to the nearest hundreth. (*Note*: For the case of $\alpha = 0.05$, $n = 50$, and $\sum_{i=1}^{50} \log x_i = -59.8$, the resulting interval is (about) (0.62, 1.08).)

(b) (4 points) Obtain a formula for a large-sample confidence interval for θ, having coverage probability approximately $1 - \alpha$, based on the asymptotic normality of the MLE of θ. Do not additionally use Slutsky's theorem — there is no need to use it, and doing so adds another source of approximation. Evaluate the confidence interval for the case of $\alpha = 0.1$, $n = 15$, and $\sum_{i=1}^{15} \log x_i = -17.94$, rounding each confidence bound to the nearest hundreth. (*Note*: For the case of $\alpha = 0.05$, $n = 50$, and $\sum_{i=1}^{50} \log x_i = -59.8$, the resulting interval is (about) (0.65, 1.16). It can be seen that this approximate interval differs from the exact one based on the same data, but there is not a big difference. However, more appreciable differences can occur if n is smaller.)

(c) (3 points) For the case of $n = 15$ and $\sum_{i=1}^{15} \log x_i = -26.836$, give the p-value that results from an exact UMP test of $H_0 : \theta \geq 1$ against $H_1 : \theta < 1$. (*Note*: You can do this by using the table of critical values for chi-square distributions which I will distribute in class. For the case of $n = 50$ and $\sum_{i=1}^{50} \log x_i = -59.249$, the resulting p-value is 0.10.)

(d) (3 points) For the case of $n = 15$ and $\sum_{i=1}^{15} \log x_i = -26.836$, give the (approximate) p-value that results from an approximate UMP test of $H_0 : \theta \geq 1$ against $H_1 : \theta < 1$, using the approximate method based on the central limit theorem which I will present in class. (*Note*: You can do this by using the table of cdf values for the standard normal distribution which I will distribute in class. For the case of $n = 50$ and $\sum_{i=1}^{50} \log x_i = -59.249$, the resulting p-value is about 0.095.)

3) (4 points) Consider the Hardy-Weinberg model and suppose that $n_1 = 32$, $n_2 = 100$ and $n_3 = 68$. Evaluate a simple large sample approximate 99% confidence interval for θ that you develop based on the fact that the MLE is consistent and asymptotically normal. (*Note*: It's okay to use results presented in the course notes — you don't have to derive everything from scratch. Also, use Slutsky's theorem to obtain an approximate pivot that will be easy to use to obtain a confidence interval for θ.)

4) (3 points) X is a random variable having pmf

$$p(x) = \begin{cases} 3\theta, & x = 0, \\ \theta, & x = 1, \\ 0.2 - 2\theta, & x = 2, \\ 0.8 - 2\theta, & x = 3. \end{cases}$$

Give the critical function of the most powerful size $\alpha = 0.05$ test of $H_0 : \theta = 0.01$ against $H_1 : \theta = 0.02$. (*Note*: The test is to be based on the single observation x from X. That is, we have a sample size of 1.)