1) We have
\[P(X < 1) = F(1-) = 1/2,\]
\[P(X = 1) = F(1) - F(1-) = 8/12 - 1/2 = 1/6,\]
\[P(1 < X < 2) = F(2) - F(1-) = \lim_{t \to 2^-}[(1/12)t + 7/12] - 1/2 = 9/12 - 1/2 = 1/4,\]
\[P(X > 1/2) = 1 - F(1/2) = 1 - 1/2 = 1/2,\]
\[P(X = 3/2) = F(3/2) - F(3/2-) = 0\] (since \(F\) is continuous at \(3/2\)),
\[P(1 < X \leq 6) = F(6) - F(1) = 1 - 8/12 = 1/3.\]

2) \[F(x) = \begin{cases} 
1, & x \geq 5 \\
10/15, & 4 \leq x < 5 \\
6/15, & 3 \leq x < 4 \\
3/15, & 2 \leq x < 3 \\
1/15, & 1 \leq x < 2 \\
0, & x < 1.
\end{cases}\]

I won’t give the sketch here, but I’ll indicate that the cdf is a right continuous step function having the same general form as the one in Fig. 4.6 on p. 161 of the text.

3) (b) We have
\[1 = p(-2) + p(0) + p(1) + p(2) = k + k + 4k + 9k = 15k,\]
which implies that \(k = 1/15\).

(c) We have
\[1 = \sum_{x=1}^{\infty} k(9)^x = (k/9)[1 + 1/9 + (1/9)^2 + (1/9)^3 + \cdots] = (k/9) \left[\frac{1}{1-(1/9)}\right] = (k/9)(9/8) = k/8,\]
which implies that \(k = 8\).

(d) We have
\[1 = \sum_{x=1}^{n} kx = k \sum_{x=1}^{n} x = k[n(n+1)/2],\]
which implies that \(k = 2/[n(n+1)]\).

4) We have that two red chips are drawn with probability
\[\binom{5}{2} \binom{5}{0} / \binom{15}{2} = 10/105 = 2/21,\]
and in this case \(X\) takes the value \(-6\) (letting \(X\) be the amount won in dollars). Similarly, we have that two white chips are drawn with probability \(2/21\), and in this case \(X\) takes the value 4, and two blue chips are drawn with probability \(2/21\), and in this case \(X\) takes the value 2. One red chip and one white chip are drawn with probability
\[\binom{5}{1} \binom{5}{1} / \binom{15}{2} = 25/105 = 5/21,\]
and in this case \(X\) takes the value \(-1\). Similarly, one red chip and one blue chip are drawn with probability \(5/21\), and in this case \(X\) takes the value \(-2\), and one white chip and one blue chip are drawn with probability \(5/21\), and in this case \(X\) takes the value 3. Altogether, the possible values of \(X\) are
\[-6, -2, -1, 2, 3, \text{ and } 4,\]
and their respective probabilities are
\[2/21, 5/21, 5/21, 2/21, 5/21, \text{ and } 2/21.\]
5) Letting \( X \) be the length of a side, and \( V = X^3 \) be the volume, the desired probability is
\[
P(V > 1.424) = P(X^3 > 1.424) = P(X > (1.424)^{1/3}) = (1.25 - (1.424)^{1/3})/(1.25 - 1) = 0.500.
\]

6) Using the fact that the possible values of \( X \) are the values at which the jump discontinuities of \( F(x) \) occur, and the associated probabilities are the magnitudes of the jumps, we have that the desired pmf is
\[
p_X(x) = \begin{cases} 1/9, & x = 6, \\ 13/45, & x = 4, \\ 1/10, & x = 2, \\ 1/2, & x = -2. \\ \end{cases}
\]
I won’t give the sketch here, but I’ll indicate that the pmf has the same general form as the one in Fig. 4.4 on p. 159 of the text.

7) The desired expectation is
\[
E(X) = \sum_{x=1}^{10} x p_X(x) = \sum_{x=1}^{10} x(1/10) = (1/10) \sum_{x=1}^{10} x = (1/10)(10)(10 + 1)/2 = 11/2,
\]
and the desired expectation is
\[
E(2\pi X) = 2\pi E(X) = 11\pi.
\]

8) Letting \( X \) be the radius of the randomly selected disk, and \( Y \) be the circumference, we have
\[
E(X) = \sum_{x=1}^{10} x p_X(x) = \sum_{x=1}^{10} x(1/10) = (1/10) \sum_{x=1}^{10} x = (1/10)(10)(10 + 1)/2 = 11/2,
\]
and the desired expectation is
\[
E(2\pi X) = 2\pi E(X) = 11\pi.
\]

9) From the cdf we can obtain that \( p(-3) = 3/8, p(0) = 3/8, \) and \( p(6) = 1/4 \). It follows that
\[
E(X) = (-3)(3/8) + (0)(3/8) + (6)(1/4) = 3/8,
\]
and
\[
E(X^2) = (-3)^2(3/8) + (0)^2(3/8) + (6)^2(1/4) = 99/8.
\]
So for the desired variance we have
\[
E(X^2) - [E(X)]^2 = 99/8 - 9/64 = 783/64 \pm 12.234,
\]
and it follows that the desired standard deviation is
\[
\sqrt{783/64} = 3\sqrt{87}/8 \approx 3.498.
\]