STAT 652: HW #5

due April 25, 2006

Instructions: Please present your solutions in order. As always, use paper which is approximately 8.5 inches by 11 inches, present tidy and easy-to-follow solutions, *draw boxes around your final answers* (but don't just give answers without supporting work), and staple all sheets together in the upper left hand corner. I don't like cover sheets, executive summaries, folders, binders, or paper clips.

I'm willing to discuss 1(a), 1(b), and 1(c) with you, but not 1(d) and 2. As usual, I'll discuss the pertinent material presented in class with you. The point values sum to 21, but I'll truncate scores at 20.

1) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x) = 2\exp(2\theta - 2x)\, I_{[\theta,\infty)}(x),$$

where $\theta \in (-\infty, \infty)$.

(a) (2 points) Give the MME of θ based on the first sample moment.

(b) (2 points) Give the mean squared error of the MME requested in part (a).

(c) (6 points) Give the Pitman estimator of θ.

(d) (5 points) Give the mean squared error of the *MLE* of θ (not the Pitman estimator). (You can make use of the following facts to avoid a somewhat messy direct derivation: (i) the MLE of θ is $X_{(1)}$; (ii) the Pitman estimator of θ is unbiased; (iii) the mean squared error of the Pitman estimator is $1/(4n^2)$. If you use all of these facts in a certain way, you can obtain the desired mean squared error without doing any integration. But if you don't see how to make good use of thse facts, it's okay to obtain the mean squared error in another manner.)

2) (6 points) Let X_1 and X_2 be iid random variables having pdf

$$f(x) = \theta^{-1}\exp(1 - x/\theta)\, I_{[\theta,\infty)}(x),$$

where $\theta \in (0, \infty)$. For the specific values $x_1 = 11.0$ and $x_2 = 5.5$, give the Pitman estimate of θ. (Note: For general n and general x_i, finding a simple expression for the Pitman estimate of θ is rather difficult. So instead of finding a general expression first, and then plugging in the given values, I suggest that you find the likelihood function based on the specific observed values which are given, and then use this likelihood to find the desired Pitman estimate. That is, plug in the x_i values at the start.)