HW 5A
STAT 544, Fall 2009

1) Letting $X$ be the amount (in dollars) that will be won, we have

$$P(X = 1.10) = 2 \frac{\binom{4}{1}}{\binom{6}{2}} = \frac{3}{7}$$

and

$$P(X = -1.00) = \frac{\binom{4}{1}\binom{1}{1}}{\binom{6}{2}} = \frac{4}{7}.$$  

(This 2nd probability could have been obtained by subtracting the 1st probability from 1.)

(a) We have

$$E(X) = (1.1)(3/7) + (-1)(4/7) = -0.1.$$  

(b) We have

$$E(X^2) = (1.1)^2(3/7) + (-1)^2(4/7) = 1.09,$$

and so

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 1.08.$$  

2) We have

$$5 = \text{Var}(X) = E(X^2) - [E(X)]^2 = E(X^2) - [1]^2,$$

which implies that $E(X^2) = 6$.

(a) We have

$$E([2 + X]^2) = E(4 + 4X + X^2) = 4 + 4E(X) + E(X^2) = 4 + 4(1) + 6 = 14.$$  

(b) We have

$$\text{Var}(4 + 3X) = 3^2\text{Var}(X) = 9(5) = 45.$$  

3) Letting $X$ be the number correct a person just guessing will get, $X$ has a binomial $(10, 1/2)$ distribution. The desired probability is $P(X \geq 7)$, which equals

$$\sum_{x=7}^{10} \binom{10}{x}(1/2)^x = (120 + 45 + 10 + 1)/2^{10} = 176/1024 = 11/64 \approx 0.172.$$