STAT 652: HW #4

due April 11, 2006

Instructions: As usual, *draw boxes around your final answers, and don't just give answers without adequate justification.* The point values sum to 21, but I'll truncate scores at 20.

I'll be willing to give you *some* assistance on parts 1(b), 1(d), 1(e), and 3(a), but don't ask me for additional hints on the other parts. Of course, I'll be glad to address any general questions about the material — just not specific questions on how to solve these problems, except for the parts for which I've indicated that I'll be willing to give you some assistance. (If I tell you how to solve them, you won't learn as much as you will if you figure them out yourself.)

Make use of results given in the class notes — it's not necessary to derive results from the notes in your solution.

1) Let $X_1, X_2, \ldots, X_n$ be iid random variables having pdf

$$f_X(x) = \theta\,(1+x)^{-(1+\theta)}\,I_{(0,\infty)}(x),$$

where $\theta > 0$.

(a) (3 points) Give the UMVUE of θ.
(b) (2 points) Give the CRLB for unbiased estimators of θ.
(c) (2 points) Give the variance of the UMVUE of θ.
(d) (3 points) Give the UMVUE of $\frac{1}{\theta}$.
(e) (2 points) Give the variance of the UMVUE of $\frac{1}{\theta}$.

2) (3.5 points) X_1 and X_2 are iid truncated Poisson random variables having pmf

$$p(x) = \frac{1}{e^\theta - 1}\,\frac{\theta^x}{x!}\,I_{\{1,2,\ldots\}}(x),$$

where $\theta > 0$. Give the UMVUE of

$$P(X_i = 1) = \frac{\theta}{e^\theta - 1}$$

based on X_1 and X_2. (For the sample $x_1 = 1$ and $x_2 = 1$, the estimate is 1. For the sample $x_1 = 1$ and $x_2 = 2$, the estimate is 1/2. For the sample $x_1 = 2$ and $x_2 = 2$, the estimate is 2/7.) You may (or may not) find the following facts useful:

(i) $\{X_1 + X_2 = t\} = \cup_{x=1}^{t-1}\{X_1 = x, X_2 = t - x\}$
(ii) $\sum_{x=0}^{t} \binom{t}{x} = 2^t$

3) Let $X_1, X_2, \ldots, X_n$ be iid $\mathrm{N}(\mu, \sigma^2)$ random variables.

(a) (3.5 points) For the case of $n = 4$, give the UMVUE of μ^2/σ. (*Hints*: You may desire to use the fact that for a sample of iid normal random variables, the sample mean and the sample variance are stochastically independent. You may also desire to use the fact that for iid $\mathrm{N}(\mu, \sigma^2)$ random variables, $\sigma^{-2}\sum_{i=1}^{n}(X_i - \overline{X})^2 \sim \chi^2_{n-1}$. Using this last fact, you might start an attack on this problem by finding $E(1/\sqrt{S^2})$.)
(b) (2 points) Give the lower bound for the variance of unbiased estimators of μ^2/σ obtained from the information inequality.