1) Let $F$ be the event the system functions, and let $C_i$ be the event that component $i$ works ($i = 1, 2, \ldots, n$). The desired probability is $P(C_1|F)$, which can be expressed (using Bayes’s formula) as

$$
\frac{P(F|C_1)P(C_1)}{P(F|C_1)P(C_1) + P(F|C_1^c)P(C_1^c)}.
$$

We have $P(C_1) = P(C_1^c) = 1/2$, and clearly $P(F|C_1) = 1$. Since $P(F|C_1^c) = 1 - P(C_1^c \cap \ldots \cap C_n^c) = 1 - (1/2)^{n-1}$, upon plugging into the Bayes’s formula expression, for the desired probability we have

$$
\frac{(1)(1/2)}{(1)(1/2) + [1 - (1/2)^{n-1}](1/2)} = \frac{1}{1 + [1 - (1/2)^{n-1}]} = \frac{1}{2 - (1/2)^{n-1}} = \frac{2^{n-1}}{2^n - 1}.
$$

Alternatively, we have

$$
P(C_1|F) = \frac{P(C_1 \cap F)}{P(F)} = \frac{P(C_1)}{P(F)} = \frac{1/2}{1 - (1/2)^n}.
$$

2) Clearly $P(X \geq 0) = 1$ (since $X$ can’t be less than 0). For $X$ to take a value of at least 1, player 1 must have the higher number (compared to player 2) in his first comparison. So, by symmetry, $P(X \geq 1) = 1/2$. For player 1 to win his first two comparisons, he needs the highest number among the first three players. So, again by symmetry, $P(X \geq 2) = 1/3$. Similarly, $P(X \geq 3) = 1/4$ and $P(X \geq 4) = 1/5$. Now clearly $P(X = 4) = P(X \geq 4) = 1/5$ (since $X$ cannot assume a value greater than 4). We also have $P(X = 3) = P(X \geq 3) - P(X \geq 4) = 1/4 - 1/5 = 1/20$, $P(X = 2) = P(X \geq 2) - P(X \geq 3) = 1/3 - 1/4 = 1/12$, $P(X = 1) = P(X \geq 1) - P(X \geq 2) = 1/2 - 1/3 = 1/6$, and $P(X = 0) = P(X \geq 0) - P(X \geq 1) = 1 - 1/2 = 1/2$. To summarize, we have

$$
P(X = 0) = 1/2, \quad P(X = 1) = 1/6, \quad P(X = 2) = 1/12, \quad P(X = 3) = 1/20, \quad P(X = 4) = 1/5.
$$