1) Letting $D$ be the event that both white dice result in the same number of spots (i.e., the white dice result in “doubles”), and $E$ be the event that the numbers on the black dice match those on the white dice, we have

$$P(E) = P(E|D)P(D) + P(E|D^C)P(D^C) = (1/36)(1/6) + (2/36)(5/6) = 11/216 \approx 0.0509.$$  

(There are 36 equally-likely outcomes of the two black dice having the form $(k_1, k_2)$, where $k_1$ is the number of spots on the 1st black die, and $k_2$ is the number of spots on the 2nd black die (where the two black dice could be viewed as being labeled, or the 1st one could be the first one to stop rolling, or the leftmost one, etc.). Similarly, there are 36 equally-likely outcomes for the two white dice. Of these 6 correspond to “doubles”, giving us that $P(D) = 6/36 = 1/6$, from which it follows that $P(D^C) = 5/6$. If the white dice results in doubles, then only one of the 36 outcomes for the black dice match the two white numbers. But if two different numbers result from the white dice, then two of the 36 possible outcomes of the black dice have numbers that match the numbers on the white dice. (E.g., if the white dice produce a 4 and a 6, then the black dice outcomes that have matching numbers are (4,6) and (6,4).))

2) Letting $C_i$ denote the event that all of the balls in set $i$ are the same color, for the desired probability we have

$$P(C_1 \cap C_2) = P(C_2|C_1)P(C_1) = \frac{3}{36} \cdot \frac{4}{12} = \left( \frac{3}{84} \right) \left( \frac{4}{220} \right) = \frac{1}{1540}.$$  

(Of the $\binom{12}{3}$ equally-likely possibilities for the 1st randomly drawn subset, 4 of them have all of the balls being the same color (since one could have the 3 amber balls, the 3 blue balls, the 3 copper balls, or the 3 green balls). The value of the conditional probability in the product above can be obtained in a similar manner, using a reduced sample space point of view.)

3) Letting $A$ be the event that a 111 is sent, and $G$ be the event that the received set of three digits result in a guess of a 111. Furthermore, let $B_{ijk}$ be the event that the received digits are, in order, $i$, $j$, and $k$. We have that $P(B_{111}|A) = (0.9)(0.9)(0.9) = 0.729$, and $P(B_{101}|A) = (0.9)(0.1)(0.9) = 0.081$, and similarly, $P(B_{101}|A)$ and $P(B_{011}|A)$ are also both equal to 0.081.

(a) For the desired probability we have (noting that the $B_{ijk}$ below are mutually exclusive)

$$P(G|A) = P(B_{111} \cup B_{101} \cup B_{101} \cup B_{001}|A) = 0.729 + 0.081 + 0.081 + 0.081 = 0.972.$$  

(Note: One does not have to express the desired probability as a conditional probability as I did, and if part (b) below did not follow next, I don’t think that I would have used a conditional probability here. But given that part (b) does follow, in order to have a more consistent use of the events in both parts, I chose to use a conditional probability here for the desired probability.)

(b) Using Bayes’s rule we have

$$P(A|B_{111}) = \frac{P(B_{111}|A)P(A)}{P(B_{111}|A)P(A) + P(B_{111}|A^C)P(A^C)} = \frac{(0.729)(0.5)}{(0.729)(0.5) + (0.001)(0.5)} = \frac{729}{730} \approx 0.99863.$$  

4) Since $A \cap B$ equals $(A \cap B \cap C) \cup (A \cap B \cap C^C)$, with the two sets in the union being disjoint, we have (using the independence of $A$ and $B$)

$$0.12 = (0.3)(0.4) = P(A)P(B) = P(A \cap B) = P(A \cap B \cap C) + P(A \cap B \cap C^C) = 0.1 + P(A \cap B \cap C^C),$$  

which implies that

$$P(A \cap B \cap C^C) = 0.12 - 0.1 = 0.02.$$  

5) Since the step function cdf (indicating a discrete random variable) has a jump discontinuity at 0, and the size of the jump is $6/7 - 2/7 = 4/7$, we have that $P(X = 0) = 4/7$. 

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**Solutions for HW 3**

**STAT 544, Spring 2015**
(b) Since the step function cdf (indicating a discrete random variable) does not have a jump discontinuity at 2/7, 2/7 is not a possible value of X, and so we have that \( P(X = 2/7) = 0 \).

(c) Since the cdf equals 6/7 on the interval [0, 3.5), and 2/7 belongs to this interval, we have that \( P(X \leq 2/7) = F_X(2/7) = 6/7 \).

(d) Since the cdf equals 6/7 on the interval [0, 3.5), and 0 belongs to this interval, we have that \( P(X \leq 0) = F_X(0) = 6/7 \), and so \( P(X > 0) = 1 - P(X \leq 0) = 1 - 6/7 = 1/7 \).

6) Since \( X \) only assumes integer values, we have that

\[
P(X \leq 16.5) = P(X \leq 16) = \sum_{x=1}^{16} \frac{x}{300} = \frac{1}{300} \sum_{x=1}^{16} x = \frac{1}{300} \left( \frac{16(16 + 1)}{2} \right) = \frac{34}{75}.
\]