Each homework assignment will be worth 20 points, and your best 10 of 12 assignment scores will be averaged to determine the homework contribution to your overall course average.

Note: Five of the ten parts below will be graded, with each graded part worth 4 points. (I won’t specify which parts will be graded until after the papers have been submitted each week.)

1) Suppose that four ordinary 6-sided fair dice will be rolled. If two of the dice are white and two of them are black, what is the probability that the numbers of spots on the two upward faces of the two black dice will be the same as the numbers of spots on the two upward faces of the white dice? (Note: If the two white dice result in 4 and 6 spots, then we’re interested in one black die resulting in 4 spots and the other black die resulting in 6 spots. If the black dice both result in a 5, then we don’t consider the numbers to be the same as those for the white dice even though the white dice result in a sum of 10 and the black dice result in a sum of 10.)

2) Consider an urn containing 3 amber balls, 3 blue balls, 3 copper balls, and 3 green balls. Suppose that a subset of 3 balls will be randomly drawn from these 12 balls, and then another subset of 3 balls will be randomly drawn from the 9 remaining balls. What is the probability that the 3 balls in the first subset drawn will be the same color and that the 3 balls in the second subset drawn will be the same color?

3) Suppose that when a binary digit is transmitted on a noisy communications channel, it is correctly received with probability 0.9, and incorrectly received (as the other binary digit) with probability 0.1. (Also assume that for each digit sent, an error will occur independently of whether or not errors occur with any other digits sent.) To improve reliability, instead of just sending a single 0 or 1, each “binary message” will be sent in triplicate, and a “majority voting scheme” will be used to interpret each set of three digits received. E.g., we know that either 000 or 111 was sent, and if we receive either 000, 001, 010, or 100, then we’ll guess that 000 was sent instead of 111, and if we receive either 111, 110, 101, or 011, then we’ll guess that 111 was sent instead of 000.

(a) If 111 is sent, what is the probability that this will be correctly guessed when a set of three digits is received?

(b) If 000 is as equally probable to be sent as 111, give the probability that 111 is sent given that 111 is received.

4) Consider events A, B, and C for which \( P(A) = 0.3, P(B) = 0.4, P(C) = 0.7, \) A and B are independent, A and C are independent, B and C are independent, and \( P(A \cap B \cap C) = 0.1 \). What is the value of and \( P(A \cap B \cap C^C) \)?

5) Consider a random variable \( X \) having cdf

\[
F_X(x) = \begin{cases} 
1, & x \geq 3.5, \\
6/7, & 0 \leq x < 3.5, \\
2/7, & -1 \leq x < 0, \\
0, & x < -1.
\end{cases}
\]

(a) What is the value of \( P(X = 0) \)?

(b) What is the value of \( P(X = 2/7) \)?

(c) What is the value of \( P(X \leq 2/7) \)?

(d) What is the value of \( P(X > 0) \)?

6) Consider a random variable \( X \) having pmf

\[
p_X(x) = \frac{x}{300} I_{\{1,2,\ldots,23,24\}}(x).
\]

Give the value of \( P(X \leq 16.5) \) (Hint: You may find it useful to use the result that the sum of the first \( n \) positive integers is \( n(n + 1)/2 \). That is, we have \( \sum_{k=1}^{n} k = n(n + 1)/2 \).)