1) Using the notation suggestion on the homework assignment, the desired probability is \( P(F_2^C \cap F_3^C \cap F_4^C) = P((F_2 \cup F_3 \cup F_4)^C) = 1 - P(F_2 \cup F_3 \cup F_4) = 1 - P(F_2) - P(F_3) - P(F_4) + P(F_2 \cap F_3) + P(F_2 \cap F_4) + P(F_3 \cap F_4) - P(F_2 \cap F_3 \cap F_4) = 1 - 3[2/3]^6 + 3[1/3]^6 - 0 = 1 - 64/243 + 1/243 - 0 = 180/243 = 20/27. 

2) There are 10! equally-likely orderings of the children. There are 5! * 5! ways to have the 5 boys to the left of the 5 girls, and 5! * 5! ways to have the 5 girls to the left of the 5 boys. So the desired probability is 2 * 5! * 5!/10! = 1/126.

3) (a) \( 3^{12} = 531,441 \)  
(b) \( \binom{12}{6} = 12!/(6!6!) = 924 \)  
(c) \( 12!/3!4!5! = 27,720 \)

4) We can imagine the 4 men lined up in alphabetical order followed by the 4 women in alphabetical order. There are 8! ways to randomly give the drinks to this arrangement of the people, but only 4! of them have the 4 men each having their own drink (since the 4 mean’s drinks have to be first in the right order, but the 4 women’s drinks can follow in any order). So the desired probability is 4!/8! = 1/1680.

5) The desired number is the number of distinguishable orderings of 3 oranges, 2 apples, and 3 scraps of paper, with the scraps of paper representing the days on which the child will get no fruit, which is \( 8!/(3!2!3!) = 560 \).

6) There are \( \binom{52}{13} \binom{39}{13} \binom{26}{13} \) ways to give the first player a subset of 13 cards of the 52, and then give the second player a subset of 13 of the remaining cards, and then give the third player a subset of the 26 remaining cards (with the fourth player then getting the 13 cards which are left). Of these ways, there are 4! of them with each player getting all of the cards of the same suit. (E.g., the first player can get the 13 spades, the second player the 13 hearts, the third player the 13 diamonds, and the fourth player the 13 clubs, but overall the four suits can be given in any of 4! orders.) So the desired probability is \( 4!/(\binom{52}{13} \binom{39}{13} \binom{26}{13}) \). Expressing each of the binomial coefficients in terms of factorials, and simplifying, yields that the probability is \( 4!(31!)/52! = 4.47 \times 10^{-28} \).

7) For two dice, we can consider a sample space consisting of 36 equally-likely ordered pairs. For the total of their top faces to be divisible by 5 it needs to be either 5 or 10. So we can consider a reduced sample space consisting of these 7 equally-likely ordered pairs: (1,4), (2,3), (3,2), (4,1), (4,6), (5,5), (6,4). Since 1 of them is (5,5), the desired (conditional) probability is 1/7.

8) Let \( B \) be the event that the bus arrives after 1:15, and let \( A \) be the event that the bus will arrive at 1:20 or later. The desired probability is

\[
P(A|B) = P(A \cap B)/P(B) = P(A)/P(B) = (10/30)/(15/30) = 10/15 = 2/3.
\]

9) Let \( A_i \) be the event that card \( i \) does not create a pair when considered with any previously selected cards. The desired probability is \( P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) \), which is equal to

\[
P(A_6|A_1A_2A_3A_4A_5)P(A_5|A_1A_2A_3A_4)P(A_4|A_1A_2A_3)P(A_3|A_1A_2)P(A_2|A_1)P(A_1).
\]

Clearly \( P(A_1) = 1 \) (since it’s impossible to get a pair with just one card). When selecting the 2nd card, there is only 1 choice that will create a pair when considered with the 1st card and 50 choices that won’t create a pair. So \( P(A_2|A_1) = 50/51 \). Given the 1st two cards do not form a pair, when selecting the 3rd card there are two cards that will pair when considered with the 1st two cards, and 48 cards that won’t. So \( P(A_3|A_1A_2) = 48/50 \). The other conditional probabilities can be obtained similarly, giving us that the desired probability is

\[
\]

10) Letting \( D_1 \) be the event that a dot is transmitted, and \( D_2 \) be the event that a dot is received, the desired (conditional) probability is \( P(D_1|D_2) \). Using Bayes’ formula, the desired probability is

\[
P(D_2|D_1)P(D_1)/(P(D_2|D_1)P(D_1) + P(D_2|D_2)P(D_2)) = (3/4)(2/5)/[(3/4)(2/5) + (1/3)(3/5)] = 3/5.
\]
11) There are 36 equally-likely outcomes we can consider for two dice, and 30 of them correspond to two
different numbers occurring. Of these, 4 correspond to a sum of 5 (we have \((1,4), (2,3), (3,2),\) and \((4,1)\)). So
using a reduced sample space point of view the desired conditional probability is \(\frac{4}{30} = \frac{2}{15} = 0.133\).

12) Letting \(W_i\) be the event that a white chip is drawn on the \(i\)th draw, and \(R_i\) be the event that a red chip
is drawn on the \(i\)th draw, the desired probability is

\[
P((W_1 \cap R_2 \cap W_3 \cap R_4) \cup (R_1 \cap W_2 \cap R_3 \cap W_4))
= P(W_1 \cap R_2 \cap W_3 \cap R_4) + P(R_1 \cap W_2 \cap R_3 \cap W_4)
= P(R_4 | W_1 R_2 W_3) P(W_3 | W_1 R_2) P(R_2 | W_1) P(W_1) + P(W_4 | R_1 W_2 R_3) P(R_3 | R_1 W_2) P(W_2 | R_1) P(R_1)
= \frac{5}{16}(\frac{8}{13})(\frac{3}{11})(\frac{5}{8}) + (\frac{8}{15})(\frac{5}{13})(\frac{5}{10})(\frac{3}{8})
= \frac{600}{18304} + \frac{600}{15600}
= 0.0712.
\]

13) Letting \(F\) be the event the first deck is selected, and \(R\) be the event that all three cards are red, the desired
(conditional) probability is \(P(F|R)\). Using Bayes theorem in a straightforward way, the desired probability equals

\[
P(F|R) = \frac{P(R|F)P(F)}{P(R)} = \frac{P(R|F)P(F) + P(R|F^C)P(F^C)}{P(R)} = \left(\frac{6}{13}\right)^2 \left(\frac{1}{2}\right) + \left(\frac{11}{13}\right)^2 \left(\frac{1}{2}\right) = \frac{23}{43}
\]

14) Using the events defined in the hints given with the statement of the problem, the desired (conditional) probability is \(P(B|U_2)\), which is equal to \(P(B \cap U_2)/P(U_2)\). In the spirit of Bayes’ formula, the multiplication
law can be applied to the numerator, yielding \(P(U_2|B)P(B)\). Now \(P(U_2|B) = 1\) since if the 2nd ball selected
is the same as the 1st, the 2nd ball is definitely used (since at the time of selecting the 2nd ball, the 1st ball was
definitely used (whether it was originally new or used)). So the numerator is just equal to \(P(B)\), which is \(1/12\)
(since there is a 1 in 12 chance of selecting the same ball). For the denominator, we can use the law of total
probability to expand \(P(U_2)\) as \(P(U_2|U_1)P(U_1) + P(U_2|U_1^C)P(U_1^C) = (1/12)(1/12) + (2/12)(11/12) = 23/144\).
So, altogether, for the desired probability we have \((1/12)/(23/144) = 12/23 \approx 0.522\).