HW 3A
STAT 544, Fall 2009

1) There are three equally likely outcomes in the reduced sample space: (4,6), (5,5), and (6,4). Two of these have at least one 6, and so the desired conditional probability is simply $2/3$.

2) Letting $W$ be the event the first two are white, and $B$ be the event the last two are black, the desired probability is

$$P(W \cap B) = P(W)P(B|W)$$

$$= \frac{\binom{5}{2}}{\binom{10}{2}} \frac{\binom{6}{2}}{\binom{13}{2}}$$

$$= \frac{36}{105} \frac{15}{78}$$

$$= \frac{6}{91}.$$ 

Alternatively, letting $W_1$ be the event the 1st ball is white, $W_2$ be the event the 2nd ball is white, $B_1$ be the event the 3rd ball is black, and $B_2$ be the event the 4th ball is black, the desired probability is

$$P(B_2|B_1W_2W_1)P(B_1|W_2W_1)P(W_2|W_1)P(W_1) = \frac{5}{12}\frac{6}{13}\frac{8}{14}\frac{9}{15} = \frac{6}{91}.$$ 

3) Letting $B$ be the event that Barbara hits the duck, and $D$ be the event that Dianne hits the duck, the desired probability is

$$P(B \cap D|B \cup D) = \frac{P(B \cap D)}{P(B \cup D)}$$

$$= \frac{P(B \cap D)}{P(B) + P(D) - P(B \cap D)}$$

$$= \frac{P(B)P(D)}{P(B) + P(D) - P(B)P(D)}$$

$$= \frac{p_1p_2}{p_1 + p_2 - p_1p_2}.$$