STAT 652: HW #2

due March 7, 2006

Instructions: Present well-organized and neat solutions. (Please present your solutions in order. For example, your solution to Problem 2 should precede your solution to Problem 3, and if a problem has two parts, part (a) should come before part (b). As always, use paper which is approximately 8.5 inches by 11 inches, present tidy and easy-to-follow solutions, draw boxes around your final answers (but don't just give answers without supporting work), and staple all sheets together in the upper left hand corner. I don't like cover sheets, executive summaries, folders, binders, or paper clips.)

Note: If I request a numerical estimate, give either the exact value, or a value which has been rounded to three signigicant digits.

Note: If I request an estimate or estimator which doesn't exist, you should *fully explain* why the estimate or estimator cannot be given.

Each part is worth 2 points, and I'll count your best 10 of 11 scores to make this a 20 point assignment. I will be willing to dole out a limited amount of advice and guidance for parts 2(a), 2(b), 2(c), 4(b), and 5 (but not for the other parts).

1) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f_X(x) = \frac{\theta\, 2^\theta}{x^{(\theta+1)}}\, I_{[2,\,\infty)},$$

where $\theta \in \Theta = (0,\,\infty)$.

(a) Give a univariate sufficient statistic for this family of distributions.

(b) Give the MLE of θ.

(c) Give the cdf of the limiting distribution of the estimator called for in part (b). (Provide justification for your answer (similar to what was called for in HW #1).)

(d) Now let the parameter space be $(1,\,\infty)$ instead of $(0,\,\infty)$, and give the MME of θ based on the first sample moment.

2) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x) = \frac{(1+\theta x)}{2}\, I_{[-1,1]}(x),$$

where $\theta \in \Theta = [-1, 1]$.

(a) Give the MME of θ based on the first sample moment.

(b) Now consider the case of $n = 1$, and give a formula for the mle of θ which results from a single observed value, x. (You should find that the mle isn't unique if $x = 0$ — more than one value of $\theta \in \Theta$ maximizes the likelihood.)

(c) Now consider the case of $n = 4$, and particular sample of values $x_1 = 0.5$, $x_2 = -0.1$, $x_3 = 0.9$, and $x_4 = -0.5$, and give a numerical value for the mle of θ, rounding to the nearest thousandth. (I suggest that you work with the likelihood, as opposed to the log-likelihood, for this problem. If you plug in the values for the x_i and multiply the factors, you'll get a fourth degree polynomial for the likelihood. To maximize this, you can set its derivative to 0 and obtain a root of the equation utilizing Newton's method, using the mme (plug into your part (a) result) as an initial value. If you do this (correctly), you'll converge to the mle in just a few iterations. (It would be nice if you also plotted the likelihood function in order to see that your solution is indeed the maximizing value of $\theta \in \Theta$.) Alternatively, you may choose to use some sort of software to obtain the mle (but if you do that, be sure to provide me with some sort of justification for your final answer).)

3) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x) = \frac{\theta + 2 - x}{2}\, I_{[\theta,\,\theta+2)}(x).$$

Give the MLE of θ.

4) Consider 100 independent trials, with each trial resulting in one of four different outcomes, according to the probabilities

$$p_1 = \theta,\ p_2 = 2\theta, p_3 = 3\theta,\ \ \&\ \ p_4 = 1 - 6\theta,$$

where $0 < \theta < 1/6$. Letting N_i $(i = 1, 2, 3, 4)$ be the number of type i outcomes that occur in a random sample of size 100, suppose it is observed that $n_1 = 6, n_2 = 33, n_3 = 45$, and $n_4 = 16$.

(a) Use a frequency substitution estimator for θ which is based on $\hat{p}_3$ (but does not incorporate $\hat{p}_1$, $\hat{p}_2$, or $\hat{p}_4$) to obtain an estimate of θ, and give the numerical value of the estimate.

(b) Give the maximum likelihood estimate of θ based on the given sample.

5) $X_1, X_2, \ldots, X_n$ are iid random variables having pdf

$$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} I_{(0,\beta]}(x),$$

where $\alpha > 0$ and $\beta > 0$. Give MLEs for α and β.