STAT 652: HW #1

Spring 2020

- For Problems 1 and 2, use either Theorem 3.4.2 or the results given in Exercise 3.32 to do parts (a) and (b). Use the mgf result concerning exponential families given in class to do part (c). Base your answer to part (d) on your answer to part (c) (i.e., identify the distribution corresponding to the mgf and simply write down the associated pdf). If you can't get the answers for Problems 1 and 2 as indicated, then some partial credit will be given for obtaining the desired answers in some other way. Give your answers in terms of θ (even though you may have converted to some other parameterization in your solution).
- For Problem 3, you're free to obtain the correct answers by whatever method you choose (other than copying someone's work), but I recommend using either Theorem 3.4.2 or the results given in Exercise 3.32.
- Problem 4 is based on my coverage of Sec. 5.3 of the text.
- To maximize partial credit, present carefully organized and neat solutions, explaining your approach and showing adequate justification for your answers. (Page 3.4.7 of the class notes has the solution for an example problem, similar to Problems 1 and 2. It will be good if your solutions have a similar style.)
- Please present your solutions in order. For example, your solution to Problem 1 should precede your solution to Problem 2, and your solution to Problem 2 should precede your solution to Problem 3. (Similarly, your solution to part (a) of a problem should come before your solution for part (b).) Use paper which is approximately 8.5 inches by 11 inches, *draw boxes around or highlight* your final answers (but don't just give answers without supporting work), and staple all sheets together in the upper *left* hand corner. *I don't like cover sheets, executive summaries, folders, binders, or paper clips.*
- If you use *Wolfram Alpha*, *Maple*, or *Mathematica* (or something similar), supply pertinent output unless the derivative or integral is rather simple. (Don't give me an "appendix" with all of your output in it. Rather, insert each computation into your homework solutions in the appropriate place.)
- While it's okay to *discuss* problems with other students, you should not copy anyone's work. (This rule applies to all homework for this class, not just this assignment.) Also, don't expect me to tell you how to solve these problems. I'll be happy to discuss similar problems, but I want you to figure out how to solve these on your own.

1) X has pdf

$$\theta x^{-(\theta+1)}\, I_{(1,\infty)}(x),$$

where $\theta > 0$. Obtain the following:

(a) (3 points) $\mathrm{E}(\log X)$,

(b) (3 points) $\mathrm{Var}(\log X)$,

(c) (2 points) the mgf of $Y = \log X$,

(d) (2 points) the pdf of $Y = \log X$ (expressed as a function of y). (Don't just indicate what type of distribution it is. Write down the density function.)

2) X has pdf

$$\frac{2}{\sqrt{\pi}\theta} \exp(-x^2/\theta^2)\, I_{(0,\infty)}(x),$$

where $\theta > 0$. Obtain the following:

(a) (3 points) $\mathrm{E}(X^2)$,

(b) (3 points) $\mathrm{Var}(X^2)$,

(c) (2 points) the mgf of $Y = X^2$,

(d) (2 points) the pdf of $Y = X^2$ (expressed as a function of y). (Don't just indicate what type of distribution it is. Write down the density function.)

3) X has pmf

$$\frac{(x-1)\theta^2(1-\theta)^{x-3}}{1+\theta}\, I_{\{3,4,5,\ldots\}}(x),$$

where $\theta \in (0,1)$. Obtain the following:

(a) (4 points) $\mathrm{E}(X)$ (in terms of θ), and also give a numerical answer (rounded to the nearest hundreth) for the case of $\theta = 0.5$,

(b) (4 points) $\text{Var}(X)$ (in terms of θ), and also give a numerical answer (rounded to the nearest hundreth) for the case of $\theta = 0.5$.

4) (4 points) X_1, X_2, and X_3 are independent random variables, with $X_k \sim \text{N}(k, k^2)$. (So X_1 has a normal distribution with mean 1 and variance 1. X_2 has mean 2 and variance 4.) Give a function of X_1, X_2, and X_3 that has an $F_{1,2}$ distribution.