STAT 554: HW #1
due Feb. 18, 2008

For this relatively simple assignment, you are to place all of your answers in the appropriate places on the answer sheet (which may be obtained from the STAT 554 web site). Do not turn in any supporting work.

You can discuss any of the problems this semester with others, except for the occassional extra credit problems that will be assigned, but you should not turn in any work that is copied from someone else, and you are not allowed to turn in any computer output which you did not generate yourself. It will be considered to be a violation of the George Mason University honor code if you turn in work which is not your own.

I will typically give you at least two weeks to do each assignment. If you don't have your paper ready to hand in the Monday that it is due, you won't be penalized if you can get it to me by 5 PM the following Thursday, except that the problems due at the time of the final exam may not be turned in late. I won't grade problems received after the deadline for the stated grace period unless I haven't started grading the others, and also you should realize that you're taking a risk unless you give me your paper in class on Monday or otherwise hand it directly to me.

In over nineteen years at GMU, to my knowledge I've never failed to successfully get a paper that someone has placed under my office door, but on rare occassions problems with a fax machine have made papers unacceptably late or missing, and so you should be aware that something might go amiss, and if for any reason I don't get your paper on time, I may not give you any credit for an assignment. If you bring your paper by to drop it off for me, the best thing is to put it under my office door (Room 150, Science & Technology 2). If locked doors at the ends of the hallway prevent you from getting to my office, I guess you'll just have to take a chance putting your paper under those doors and e-mailing and calling me to let me know where to go looking for your paper.

Always send me an e-mail message to notify me if you fax your paper to me, or if you drop your paper off for me at my office. I won't go looking for your paper or fax unless you notify me about it, and if (for whatever reason) I don't get your paper to grade by the time I start grading the others, your paper may not be graded. (The fax number is (703) 993-1700, but many people share the same fax machine, so you should clearly indicate that your fax is for me on a cover sheet.) For a variety of reasons, *don't attempt to send me your homework via e-mail*. Of course, *the easiest and safest scheme will be to simply bring your papers to class on the Mondays that they are due.*

The maximum score on each homework assignment will range from about 10 to 40 points, but the points for the various parts may total more than the stated maximum score. In such a case you don't have to worry yourself sick if you can't get one or two of the parts. (In return for this generousity on my part, I'll ask that you don't argue with me about the amount of partial credit you receive for an incorrect answer. Of course, you should bring the matter to my attention if I make an obvious mistake like adding up your points incorrectly. But as far as partial credit goes, I'll work very hard to grade everyone's papers consistently, and so it won't be fair if I change the partial credit on your paper unless I make sure that I change the partial credit on everyone else's paper who made the same mistake.)

By the end of the semester you will have been given about 120 points worth of HW problems (not counting the extra credit problems and "bonus points" — if we count those, then it'll be at least 130), and I'll truncate your homework total at 100 points. So, all in all, you can miss quite a few parts and still get a perfect HW score to average into your overall course score. I'll warn you now that a lot of the points will be offered to you towards the end of the semester, and so it may be wise to strive for good scores on the first 3 assignments, even though they won't count as much, so that you'll give yourself a bit of a cushion at the end when you may not have time to do a careful job on all of the problems.

For this assignment, parts 1(d), 1(g), 1(i), 1(j), 1(n), 2(b), 2(d), and 3(b) are worth 0 points since I'm supplying you with the answers to them (in order to help you make sure that you understand this assignment). All of the other parts are each worth 1 point. The maximum score for this assignment is 12 (even though there are 14 points worth of problems).

Round all answers as indicated on the answer sheet (sometimes rounding to the nearest hundreth and other times rounding to the nearest thousandth).

1) When paying roulette, in order for a bet on the outcome black to have a positive expected net profit, the

probability of the outcome black must exceed 0.5. This problem deals with hypothesis tests of $H_0 : p \leq 0.5$ against $H_1 : p > 0.5$, where p is the probability of a black outcome, and iid Bernoulli trials are to be assumed. Letting y be the observed number of black outcomes in n trials, consider tests which reject the null hypothesis in favor of the alternative whenever y is sufficiently large.

(a) For the case of $n = 40$, *give the largest upper-tail rejection region which is possible for a (nonrandomized) level 0.01 test.*

(b) *Give the size of the test corresponding to rejection region called for in part (a).* (*Note*: Obviously, since we are dealing with a level 0.01 test, the size cannot exceed 0.01.)

(c) *Give the power of the test corresponding to rejection region called for in part (a) if* $p = 0.6$. (*Note*: I'll talk about *power* during the third class, but you can read about it ahead of time in Sec. 1.4 of the class notes.)

(d) *Give the power of the test corresponding to rejection region called for in part (a) if* $p = 0.7$.

** For parts (e) through (h), repeat parts (a) through (d) using $n = 80$ (instead of $n = 40$). (Note that for a given value of p, the power of the level 0.01 test increases as n is increased.)

** For parts (i) through (l), repeat parts (a) through (d) using $n = 120$ (instead of $n = 40$).

(m) For the case of $n = 40$, *describe the rejection criterion for a randomized size 0.01 upper-tail test.* (*Note*: You can examine my answer to part (n), which is given on the answer sheet for this assignment, to see what type of an answer I'm looking for here.)

(n) For the case of $n = 80$, *describe the rejection criterion for a randomized size 0.01 upper-tail test.*

2) Suppose that you are one of the fortunate few who receive an invitation to Cliff's Casino Night, a gala event where I get out all of my gambling paraphernalia and transform my humble home into a lavishly equipped betting parlor having keno, craps, blackjack, and roulette. If you are a cautious and even only a slightly suspicious person, you should decline the complimentary rotgut drinks and decide to carefully observe my activities.

Since I have noticed in the past that more people tend to bet on red than on black, I have altered my roulette wheel so that red will occur with a smaller probability than would be the case than if it were an "ideal wheel" (thus letting me enjoy an expected profit which exceeds the usual house advantage — if I were to notice that a particular crowd favored betting on black, I can spill food onto the initial roulette wheel and replace it with one which favors red over black). If you observe only 10 outcomes of red in 30 trials, would you be able to claim that there is statistically significant evidence to reject $H_0 : p \geq 9/19$ in favor of $H_1 : p < 9/19$, where p is the probability of observing the outcome red?

(a) Respond to this query by *reporting the p-value which results from the appropriate test.*

Note that there is no need to use the normal approximation for part (a) since Minitab or similar software can be used to obtain an exact answer based on the binomial distribution. Nevertheless, if one does use the approximation, with the continuity correction, you'll see that it does okay, yielding an approximate p-value which matches the exact p-value when both are rounded to the nearest hundreth (and isn't off by much when values are rounded to the nearest thousandth).

(b) *Approximate the p-value requested in part (a) using the normal approximation with the continuity correction.*

(c) *Approximate the p-value using the normal approximation without the continuity correction*, and note that this approximation isn't as good in this case (not even matching the exact value when both are rounded to the nearest hundredth). (*Note*: Approximations are commonly used in statistics, but it's often important to use the best possible approximations when an exact computation isn't feasible. It's also important to check to see that any guidelines or rules of thumb for the approximations are satisfied. In part (e), even the approximation using the continuity correction fails to produce an answer with even one correct significant digit even though the sample size is 74, because 74 just isn't large enough given that we are working with a rather small value of p.)

I will also have another roulette wheel that has 37 numbers on it $(0, 1, 2, \ldots, 35, 36)$. Since I have observed a tendency for people not to bet on the number 13, I have rigged this wheel so that 13 comes up more often than the other numbers do. If you observe 5 outcomes of 13 in 74 trials, would you be able to claim that there is statistically significant evidence to reject $H_0 : p \leq 1/37$ in favor of $H_1 : p > 1/37$, where p is the probability of observing the outcome 13?

(d) Respond to this query by *reporting the p-value which results from the appropriate test.* (*Note*: You should get a value which does not lead to a statistically significant implication that I'm a crook (but it's *very* close), if a significance level of 0.05 is employed.)

(e) *Approximate the p-value requested in part (d) using the normal approximation with the continuity correction.*

3) In order to hopefully get a better understanding of the concept of a consistent estimator, consider iid Normal(μ, 9) random variables $X_1, X_2, \ldots, X_n$, and determine the value of $P(|\overline{X} - \mu| \leq 0.3)$ for the values of n indicated below. (*Note*: $\overline{X}$ is a consistent estimator of μ, and the probability of it assuming a value within 0.3 of the estimand increases as n increases.)

(a) $n = 10$

(b) $n = 100$

(c) $n = 1000$

A just-for-fun problem (not to be turned in for credit): Let Y be a binomial(n, p) random variable. Show that $P(Y \geq k)$ is an increasing function of p. *Hint*: Differentiate $P(Y \geq k)$ w.r.t. p. Consider two cases: $k \geq np$ and $k < np$. In the first case, the derivative is clearly nonnegative, while in the second case it is no smaller than

$$\frac{1}{p(1-p)} \sum_{j=0}^{n} (j - np) \binom{n}{j} p^j (1-p)^{n-j} = \frac{E(Y - np)}{p(1-p)} = 0.$$

A just-for-fun problem (not to be turned in for credit): Show that S^2 is an unbiased estimator for the variance of any distribution (for which the variance exists).

Extra Credit Problem

For the case of normal distributions, find an unbiased estimator for the standard deviation. (You might consider modifying S by multiplying it by a constant (which might be a function of n).)

Note: This semester, extra credit problems will be worth one point apiece, unless it is stated otherwise. Some of these problems are a tad tricky and may require that you use results which are not explicitly given in class (but all of them can be done using material from a good probability class, the class notes for STAT 554, and possibly some indicated parts of Miller's *Beyond ANOVA*.). Since the maximum score on each homework assignment is typically less than the total number of points for the regular problems on the assignment, working the extra credit problems will only benefit your grade if you screw up on some of the regular problems of the assignment (and my guess is that most people who are able to do most of the extra credit problems should be able to get an A in STAT 554 even if they don't turn in any extra credit problems). With regard to your grade in STAT 554, you'll probably be better off if you spend time checking over the regular parts of the assignments and not spend a great deal of time worrying about the extra credit problems. But if you plan to take additional courses in statistics, any effort you put into these extra credit problems may pay off later on. *If you plan to turn in any extra credit problems for credit, you should solve them entirely on your own. (I might give you a tiny hint on some of the problems, but otherwise the honor code applies.) Always put any extra credit solutions at the end of your paper, starting them on a new sheet that has your name on it in case I have to tear these problems off in order to save them to grade at a later time.*