STAT 652: HW #1

due Feb. 14, 2006

Instructions: There are 6 parts to this assignment, each is worth 2 points, and I'll count your best 5 parts. (So the maximum score on this assignment is 10.)

To maximize partial credit, present carefully organized and neat solutions. Please present your solutions in order. For example, your solution to Problem 1 should precede your solution to Problem 2, and your solution to Problem 2 should precede your solution to Problem 3. Use paper which is approximately 8.5 inches by 11 inches, present tidy and easy-to-follow solutions, *draw boxes around or highlight* your final answers (but don't just give answers without supporting work), and staple all sheets together in the upper *left* hand corner. *I don't like cover sheets, executive summaries, folders, binders, or paper clips.*

For each part you are asked to consider a sequence of random variables, and give either the cdf, the pdf, or the pmf of the limiting distribution. (Be sure to give me the form of the limiting distribution that I request. That is, don't give the pmf if I request the cdf.) In one or more cases, it may be that a limiting distribution does not exist, and in such cases you cannot supply me with what I'm requesting. In such cases you should state that a limiting distribution does not exist and justify your conclusion.

Note

Don't ask me for any help on 1(b), 3, and 4. (You can discuss these problems amongst yourselves to some extent, but don't copy someone's solution or let anybody copy your solution, and don't give anyone so many hints that you've practically solved the problem for them.)

I am willing to discuss 1(a), 2, and 5 with you (but don't expect me to show you how to do any of these completely (I'm only willing to look over your work and offer small suggestions)). Of course, you can discuss general concepts with me during office hours. I don't like to discuss too much about your specific homework problems because doing so ruins the educational value of them.

Hints: You *may* desire to use the following facts on one or more of the problems.
(i) If $Z \sim N(0,1)$, then $Z^2 \sim \chi^2_1$.
(ii) If $|x| < 1$, then $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$.

1) $X_1, X_2, \ldots$ are iid random variables having pdf

$$f_X(x) = \frac{2}{x^3}\, I_{(1,\infty)}.$$

(a) Let

$$T_n = \max\{X_1, \ldots, X_n\}/\sqrt{n}.$$

Consider the sequence $T_1, T_2, \ldots$ and give the pmf or pdf of the limiting distribution.

(b) Now let $T_n = \sqrt{n}\min\{X_1, X_2, \ldots, X_n\}$. Consider the sequence $T_1, T_2, \ldots$ and give the pmf or the pdf of the limiting distribution.

2) T_n is a random variable having pdf

$$f_n(t) = \begin{cases} n, & 0 < t < n^{-1}, \\ 0, & \text{otherwise.} \end{cases}$$

Consider the sequence $T_1, T_2, \ldots$ and give the pdf or the pmf of the limiting distribution. (It can be noted that $\lim_n f_n(t)$ is not equal to the pdf or pmf of the limiting distribution which is requested. This shows that $\lim_n f_n(t)$ is not guaranteed to be the pdf or pmf of the limiting distribution. (So to determine a limiting distribution, it doesn't always work to take the limit of $f_n(t)$.) However, if T and $T_1, T_2, \ldots$ are all continuous random variables and $\lim_n f_n(t) = f_T(t)$ for almost all t, then it can be concluded that $T_1, T_2, \ldots$ converges in distribution to T (and so $f_T(t)$ is the pdf of the limiting distribution). Similarly, for discrete random variables, the limit of the sequence of pmf's does *not always* yield the pmf of the limiting distribution (but in some cases it is true that the limit of the pmf's is the pmf of the limiting distribution).)

3) $X_1, X_2, \ldots$ are iid exponential random variables, for which $E(X_i) = 5$, and

$$T_n = \sqrt{\frac{\sum_{i=1}^n X_i^2}{n-1}}.$$

Consider the sequence $T_1, T_2, \ldots$ and give the cdf of the limiting distribution.

4) V_n is a Poisson $(\sqrt{n})$ random variable, and $T_n = V_n/\sqrt{n}$. Consider the sequence $T_1, T_2, \ldots$ and give the cdf of the limiting distribution.

5) $X_1, X_2, \ldots$ are iid Bernoulli $(1/4)$ random variables, and

$$T_n = \frac{4\sum_{i=1}^n X_i \; - \; n}{\sqrt{n}}.$$

Consider the sequence $T_1, T_2, \ldots$ and give the pmf or pdf of the limiting distribution.

Comments

Some of you may find this assignment to be rather difficult. But if you can manage to get at least 3 parts done correctly, that should be okay for this assignment. If you get stuck, try:

(i) breaking the problem up into smaller parts;

(ii) exploring more than one method of attack.

Don't forget about mgfs, but you don't have to use them on every problem (and I don't think that you should try to use them on every problem). Often, a simple direct attack is all that is needed. Don't forget about the law of large numbers, the central limit theorem, Slutsky's theorem, the definition of convergence in probability, and other powerful results covered in the class notes. (If you encounter a sum of *iid* random variables, then you may first want to consider using either the law of large numbers or the central limit theorem, but of course some other method of attack may also work, and may even work better. It's just that I would typically consider investigating one of these limit laws first. (Note that I stipulated that the random variables in the sum are iid.))

You should be able to handle these problems using the material on pages B-6 to B-22 of the class notes and suitable knowledge of probability. Feel free to use probability distribution facts given in the material that I supplied you with. (That is, you don't have to derive those probability facts as part of your solutions.)

Please review comments pertaining to late homework and getting help from others that were given on the syllabus.