HW 12A
STAT 544, Fall 2009

1) For \( x > 0 \),

\[
f_X(x) = \int_0^x \frac{2e^{-2x}}{x} \, dy = 2e^{-2x},
\]

and

\[
f_{Y|X}(y|x) = \frac{(2e^{-2x}/x) I_{[0,x]}(y)}{2e^{-2x}} = \frac{1}{x} I_{[0,x]}(y).
\]

So, for the desired conditional expectation we have

\[
E(Y|X = x) = \int_0^x \frac{y}{x} \, dy = \frac{y^2}{(2x)}|_0^x = x/2.
\]

2) Using (iv) of Proposition 4.2 on p. 323 of Ross, we have that

\[
\text{Cov}(Y_n, Y_{n+1}) = \text{Cov}(X_n + X_{n+1} + X_{n+2}, X_{n+1} + X_{n+2} + X_{n+3})
\]

\[
= \text{Cov}(X_n, X_{n+1}) + \text{Cov}(X_n, X_{n+2}) + \text{Cov}(X_n, X_{n+3}) + \text{Cov}(X_{n+1}, X_{n+2}) + \text{Cov}(X_{n+1}, X_{n+3}) + \text{Cov}(X_{n+2}, X_{n+3})
\]

\[
= 0 + 0 + \sigma^2 + 0 + 0 + \sigma^2 + 0
\]

\[
= 2\sigma^2.
\]

3) Letting

\[
Y = \begin{cases} 
1, & \text{if a type 1 bulb is selected,} \\
2, & \text{if a type 2 bulb is selected,}
\end{cases}
\]

we have \( p_Y(1) = p_Y(2) = 0.5 \), so that \( E(Y) = 1.5 \). Since \( E(X|Y) = \mu_Y = Y \), it follows that

\[
E(X) = E(E(X|Y)) = E(Y) = 1.5.
\]