1) Since there are \( n \) urns and \( n \) balls, the only way that none of the urns can be empty is for each urn to contain exactly one ball. Since the only ball that can possibly go into urn \( n \) is ball \( n \), if none of the urns are to be empty, ball \( n \) must go into urn \( n \). Since the only balls that can go into urn \( n - 1 \) are balls \( n - 1 \) and \( n \), if none of the urns are to be empty, ball \( n - 1 \) must go into urn \( n - 1 \) (since it’s already been established that if none of the urns are to be empty, ball \( n \) must go into urn \( n \)). Proceeding in a likewise manner, we can conclude that if none of the urns are to be empty, it must be that ball \( i \) must go into urn \( i \), for \( i = 1, 2, \ldots, n \).

So, letting \( A_i \) denote the event that ball \( i \) goes into urn \( i \), the desired probability is

\[
P(\cap_{i=1}^{n} A_i) = \prod_{i=1}^{n} P(A_i) = \prod_{i=1}^{n} \frac{1}{i} = \frac{1}{n!}.
\]

2) Letting \( A_j \) denote the event that the \( j \)th person gets a card matching his/her age, and noting that exactly one of the 1000 cards will have the number matching any of the people’s ages, we have \( P(A_j) = 1/1000 \), for \( j = 1, 2, \ldots, 1000 \). Letting \( X \) be the number of matches, and

\[
I_j = \begin{cases} 
1, & \text{if } A_j \text{ occurs,} \\
0, & \text{otherwise,}
\end{cases}
\]

we have that the desired expectation is

\[
E(X) = E \left( \sum_{j=1}^{1000} I_j \right) = \sum_{j=1}^{1000} P(A_j) = \sum_{j=1}^{1000} \frac{1}{1000} = 1.
\]