Additional Important Problems

1) If X has mgf

$$M_X(t) = \frac{1}{2 - e^t} \qquad (t < \log 2),$$

give the value of $E(X)$.

answer: 1

2) Using the joint pdf,

$$f(x, y) = \begin{cases} 2xe^{-y}, & x > 0,\ y > x^2, \\ 0, & \text{otherwise}, \end{cases}$$

obtain each of the following:
(a) the marginal pdf of X,
(b) the marginal pdf of Y,
(c) the conditional pdf of X given $Y = 9$,
(d) the conditional expectation of X given $Y = 9$,
(e) $Cov(X, Y)$,
(f) the variance of $X + Y$,
(g) the pdf of $V = X^2/Y$.

answers:
(a) $2xe^{-x^2}\, I_{(0,\,\infty)}(x)$
(b) $ye^{-y}\, I_{(0,\,\infty)}(y)$
(c) $\frac{2}{9}x\, I_{(0,\,3)}(x)$
(d) 2
(e) $\frac{\sqrt{\pi}}{4} \doteq 0.443$
(f) $3 - \frac{\pi}{4} + \frac{\sqrt{\pi}}{2} \doteq 3.101$
(g) $I_{(0,\,1)}(v)$ (pdf of a uniform (0, 1) random variable)

3) Using the joint pdf,

$$f(x, y) = \begin{cases} \frac{6x^3 + 2xy}{5}, & 0 < x < 1,\ 0 < y < 2, \\ 0, & \text{otherwise}, \end{cases}$$

obtain each of the following:
(a) the cdf of $V = \max\{X, Y\}$,
(b) the cdf of $W = \min\{X, Y\}$.

answers:
(a)

$$F_V(v) = \begin{cases} 1, & v \geq 2, \\ \frac{3v + v^2}{10}, & 1 \leq v < 2, \\ \frac{3v^5 + v^4}{10}, & 0 < v < 1, \\ 0, & v \leq 0. \end{cases}$$

(b)

$$F_W(w) = \begin{cases} 1, & w \geq 1, \\ \frac{3w + 5w^2 + 5w^4 - 3w^5}{10}, & 0 < w < 1, \\ 0, & w \leq 0. \end{cases}$$

4) Consider independent random variables, X and Y, where X is a uniform $(0,\, 2)$ random variable, and Y has pdf

$$f_Y(y) = \frac{y}{2}\, I_{(0,\,2)}(y),$$

and obtain each of the following:

(a) the pdf of $V = \max\{X, Y\}$,

(b) the pdf of $W = \min\{X, Y\}$.

answers:

(a)

$$f_V(v) = \frac{3}{8} v^2\, I_{(0,\,2)}(v)$$

(b)

$$f_W(w) = \frac{4 + 4w - 3w^2}{8}\, I_{(0,\,2)}(w)$$