Solutions for Additional Extra Ch. 5 Problems

1) We have

$$E(X^4) = \int_{-\infty}^{\infty} x^4 f_X(x)\,dx = \int_0^1 x^4\,dx = (x^5/5)|_0^1 = 1/5.$$

2) Since $P(X < x) = P(X \le x) = \Phi((x-\mu)/\sigma) = \Phi((x-10)/6)$, we have
(a) $P(X < 2) = \Phi((2-10)/6) = \Phi(-4/3) \doteq 0.091$,
(b) $P(X > 18) = 1 - \Phi((18-10)/6) = 1 - \Phi(4/3) \doteq 0.091$,
(c) $P(10 < X < 18) = \Phi((18-10)/6) - \Phi((10-10)/6) = \Phi(4/3) - \Phi(0) \doteq 0.909 - 0.5 = 0.409$.

3) Letting F denote the cdf of X, it can be shown that on the support of X,

$$F(x) = 1 - 10/x.$$

So for $x \in (0,1)$ we have

$$x = F(F^{-1}(x)) = 1 - 10/[F^{-1}(x)],$$

which gives us that for $x \in (0,1)$,

$$F^{-1}(x)) = 10/(1-x).$$

So the desired function of U is $10/(1-U)$. (*Note*: Because U and $1-U$ have the same distribution, $10/U$ will also work.)

4) Letting X be the number of unacceptable items in the next 150, we have that X is a binomial (150, 0.05) random variable. The desired probability is

$$P(X \le 10) \simeq \Phi([10 + 1/2 - 150(0.05)]/\sqrt{150(0.05)(0.95)}) \doteq \Phi(0.1.123903) \doteq 0.869.$$