Solutions for Extra Ch. 4 Problems

1) Letting X be the amount (in dollars) that will be won, we have

$$P(X = 1.10) = 2\frac{\binom{4}{2}}{\binom{8}{2}} = 3/7$$

and

$$P(X = -1.00) = \frac{\binom{4}{1}\binom{4}{1}}{\binom{8}{2}} = 4/7.$$

(This 2nd probability could have been obtained by subtracting the 1st probability from 1.)

(a) We have

$$E(X) = (1.1)(3/7) + (-1)(4/7) = -0.1.$$

(b) We have

$$E(X^2) = (1.1)^2(3/7) + (-1)^2(4/7) = 1.09,$$

and so

$$Var(X) = E(X^2) - [E(X)]^2 = 1.08.$$

2) We have

$$5 = Var(X) = E(X^2) - [E(X)]^2 = E(X^2) - [1]^2,$$

which implies that $E(X^2) = 6$.

(a) We have

$$E([2 + X]^2) = E(4 + 4X + X^2) = 4 + 4E(X) + E(X^2) = 4 + 4(1) + 6 = 14.$$

(b) We have

$$Var(4 + 3X) = 3^2\,Var(X) = 9(5) = 45.$$

3) Letting X be the number correct a person just guessing will get, X has a binomial (10, 1/2) distribution. The desired probability is $P(X \geq 7)$, which equals

$$\sum_{x=7}^{10} \binom{10}{x} (1/2)^{10} = (120 + 45 + 10 + 1)/2^{10} = 176/1024 = 11/64 \doteq 0.172.$$

4) Letting X be the number of digits incorrectly received, X has a binomial (10, 1/5) distribution. The desired probability is $P(X \geq 3)$, which equals

$$\sum_{x=3}^{5} \binom{5}{x} (1/5)^x (4/5)^{5-x} = 181/3125 \doteq 0.0579.$$

5) Letting X be the number of heads in the 10 flips, X has a binomial (10, p) distribution, where p is either 0.4 or 0.7. Letting A be the event that $X = 7$, and B be the event that the coin 1 is chosen, the desired probability is

$$P(A) = P(A|B)P(B) + P(A|B^C)P(B^C) = [P(A|B) + P(A|B^C)](1/2),$$

which equals

$$\left[\binom{10}{7}(0.4)^7(0.6)^3 + \binom{10}{7}(0.7)^7(0.3)^3\right](1/2) \doteq 0.155.$$

6) Let X be a geometric (1/38) random variable.

(a) The desired probability is $P(X > 5) = (37/38)^5 \doteq 0.875$.

(b) The desired probability is $P(X = 4) = (37/38)^3(1/38) \doteq 0.0243$.

7) Since $Var(2X) = 4\,Var(X)$, the only way $Var(2X)$ can equal $2\,Var(X)$ is if $Var(X) = 0$. Since $Var(X) = p(1 - p)$, p must equal either 0 or 1.